

4.8 Use the Quadratic Formula and the Discriminant

Complete the square of the standard form quadratic:

$$\cancel{a}x^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x + \frac{4b^2}{4a^2} = \frac{4a}{4a} \frac{c}{a} + \frac{4b^2}{4a^2}$$

$$-\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXAMPLE 1 Solve an equation with two real solutionsSolve $x^2 + 3x = 2$.

$$x^2 + 3x - 2 = 0$$
$$a = 1 \quad b = 3 \quad c = -2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 + 8}}{2} = \boxed{\frac{-3 \pm \sqrt{17}}{2}}$$

EXAMPLE 2 Solve an equation with one real solution

$$\text{Solve } 25x^2 - 18x = 12x - 9.$$

~~$-12x + 9$~~ ~~$-12x + 9$~~

$$25x^2 - 30x + 9 = 0$$

$$a = 25 \quad b = -30 \quad c = 9$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{30 \pm \sqrt{900 - 4(25)(9)}}{2(25)}$$

$$x = \frac{30 \pm \sqrt{900 - 900}}{50} = \frac{30 \pm 0}{50} = \frac{30}{50} = \boxed{\frac{3}{5}}$$

EXAMPLE 3 Solve an equation with imaginary solutionsSolve $-x^2 + 4x = 5$.

$$-x^2 + 4x - 5 = 0$$

$$\begin{aligned} a &= -1 \\ b &= 4 \\ c &= -5 \end{aligned}$$

$$x = \frac{-4 \pm \sqrt{16 - 4(-1)(-5)}}{2(-1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 20}}{-2} = \frac{-4 \pm \sqrt{-4}}{-2}$$

$$x = \frac{-4 \pm 2i}{-2} = \boxed{2 \pm i}$$

EXAMPLE 4 Use the discriminant

Find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

a. $x^2 - 8x + 17 = 0$

$$b^2 - 4ac$$

$$(-8)^2 - 4(1)(17)$$

$$64 - 68$$

-4

2 img.

b. $x^2 - 8x + 16 = 0$

$$(-8)^2 - 4(1)(16)$$

$$64 - 64$$

0

1 real

c. $x^2 - 8x + 15 = 0$

$$(-8)^2 - 4(1)(15)$$

$$64 - 60$$

4

2 real