

4.5 Solve Quadratic Equations by Finding Square Roots

EXAMPLE 1 Use properties of square roots

Simplify the expression.

a. $\sqrt{80}$

$$\sqrt{16} \sqrt{5}$$
$$\boxed{4\sqrt{5}}$$

b. $\sqrt{6} \cdot \sqrt{21}$

$$\sqrt{6 \cdot 21}$$
$$\sqrt{126}$$
$$\sqrt{9} \sqrt{14}$$
$$\boxed{3\sqrt{14}}$$

c. $\sqrt{\frac{4}{81}}$

$$\frac{\sqrt{4}}{\sqrt{81}}$$
$$\boxed{\frac{2}{9}}$$

EXAMPLE 2 Rationalize denominators of fractions

Simplify

a. $\frac{\sqrt{5}}{2}$

$$\frac{\sqrt{5}}{2} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{\sqrt{10}}{2}$$

$\frac{\sqrt{17}}{\sqrt{12}} = \frac{\sqrt{17}}{\sqrt{2}}$

$\frac{\sqrt{17}}{\sqrt{4}\sqrt{3}} = \frac{\sqrt{17}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\frac{\sqrt{51}}{2 \cdot 3} = \frac{\sqrt{51}}{6}$

EXAMPLE 2 Rationalize denominators of fractions

Simplify

$$(a+b)(a-b)$$

$$a^2 - \cancel{ab} + \cancel{ab} - b^2$$

$$\boxed{a^2 - b^2}$$

EXAMPLE 2 Rationalize denominators of fractions

Simplify

b. $\frac{3(7-\sqrt{2})}{(7+\sqrt{2})(7-\sqrt{2})}$

$$\frac{21-3\sqrt{2}}{49-7\sqrt{2}+7\sqrt{2}-2}$$

$$(7)^2 - (\sqrt{2})^2 \rightarrow \frac{21-3\sqrt{2}}{49-2} = \boxed{\frac{21-3\sqrt{2}}{47}}$$

EXAMPLE 2 Rationalize denominators of fractions

Simplify

$$\frac{-6(7+\sqrt{5})}{(7-\sqrt{5})(7+\sqrt{5})}$$

$$\frac{-42-6\sqrt{5}}{(7)^2-(\sqrt{5})^2}$$

$$\frac{-42-6\sqrt{5}}{49-5}$$

$$\frac{-42-6\sqrt{5}}{44}$$

$$\frac{\cancel{2}(-21-3\sqrt{5})}{\cancel{2}(22)}$$

$$\frac{-21-3\sqrt{5}}{22}$$

$$\frac{2}{4+\sqrt{11}} \cdot \frac{4-\sqrt{11}}{4-\sqrt{11}}$$

$$= \frac{8-2\sqrt{11}}{16-11}$$

$$= \frac{8-2\sqrt{11}}{5}$$

EXAMPLE 3 Solve a quadratic equation

$$\text{Solve } 3x^2 + 5 = 41$$

$$\frac{3x^2}{3} = \frac{36}{3}$$

$$\sqrt{x^2} = \sqrt{12}$$

$$x = \pm \sqrt{12}$$

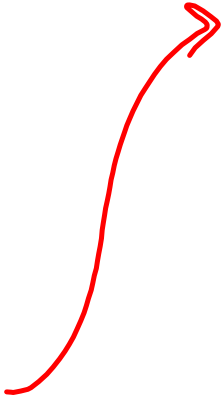
$$x = \pm 2\sqrt{3}$$

EXAMPLE 4

What are the solutions of the equation $\frac{1}{5}(z+3)^2 = 7$?

$$\begin{aligned} 5 \left(\frac{1}{5}(z+3)^2 \right) &= 7(5) \\ \sqrt{(z+3)^2} &= \sqrt{35} \\ z+3 &= \pm \sqrt{35} \\ -3 &\quad -3 \\ z &= \pm \sqrt{35} - 3 \end{aligned}$$

$-3 \pm \sqrt{35}$



SOLVING QUADRATIC EQUATIONS Solve the equation.

$$\begin{aligned} (27.) \quad & \frac{-3w^2}{-3} = \frac{-213}{-3} \\ & \sqrt{w^2} = \sqrt{71} \\ & w = \pm\sqrt{71} \end{aligned}$$