4.6 Perform Operations with Complex Numbers

What *squared* is -4?

$(2)^2 = 4$

$(-2)^2 = 4$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm\sqrt{-4}$

$\pm\sqrt{4}\ \sqrt{-1}$

$x = \pm 2i$

$i = \sqrt{-1}$

EXAMPLE 1 **Solve a quadratic equation**

Solve the equation.

$2x^2 + 11 = -37.$

$-11 \quad -11$

$\frac{2x^2}{2} = \frac{-48}{2}$

$\sqrt{x^2} = \sqrt{-24}$

$x = \pm\sqrt{4}\sqrt{6}\sqrt{-1}$

$x = \pm 2\sqrt{6}\,i$

$x = \pm 2i\sqrt{6}$

$3x^2 - 7 = -31$

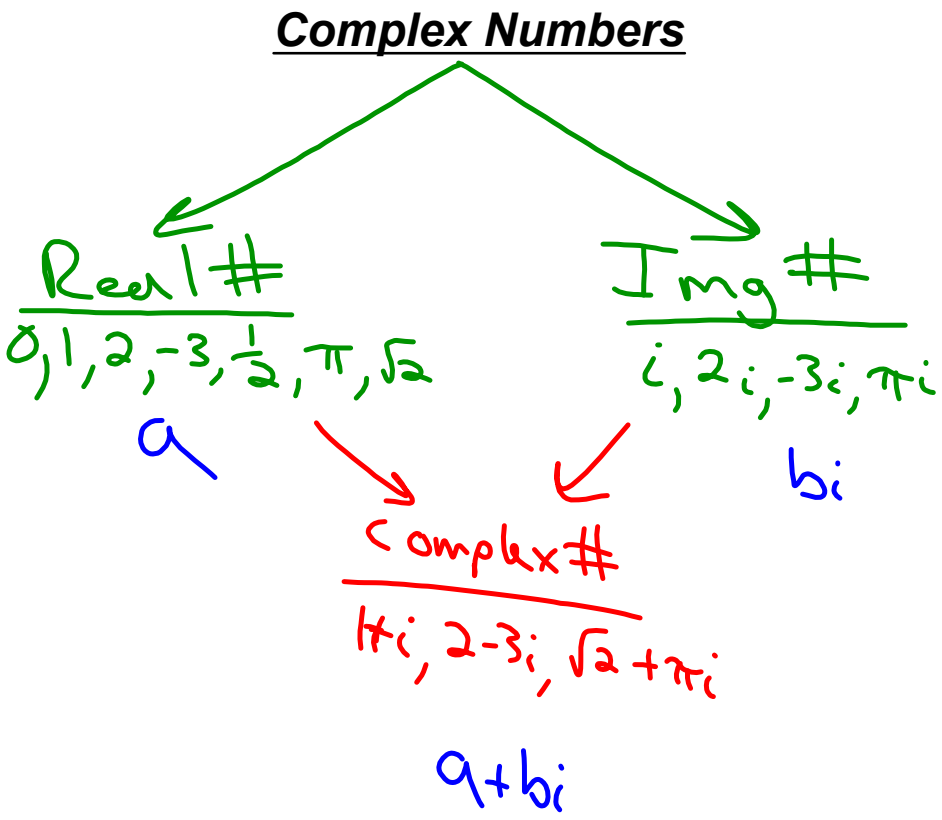
$3x^2 = -24$

$\sqrt{x^2} = \sqrt{-8}$

$x = \pm\sqrt{-8}$

$\sqrt{4}\ \sqrt{2}\ \sqrt{-1}$

$x = \pm 2i\sqrt{2}$


Complex Numbers
Real #
0, 1, 2, -3, 1/2, π, √2
Img #
i, 2i, -3i, πi
a
bi
Complex #
1+i, 2-3i, √2+πi
a+bi

EXAMPLE 2 **Add and subtract complex numbers**

Write the expression as a complex number in standard form.

a. $(8 - i) + (5 + 4i)$

$8 - i + 5 + 4i$

$13 + 3i$

b. $(7 - 6i) - (3 - 6i)$

$7 - 6i - 3 + 6i$

4

$4 + 0i$

c. $10 - (6 + 7i) + 4i$

$10 - 6 - 7i + 4i$

$4 - 3i$

EXAMPLE 4 **Multiply complex numbers**

Write the expression as a complex number in standard form.

a. $4i(-6 + i)$

$-24i + 4i^2$

$-24i + 4(-1)$

$-4 - 24i$

b. $(9 - 2i)(-4 + 7i)$

$-36 + 63i + 8i - 14i^2$

$-36 + 71i - 14(-1)$

$-36 + 71i + 14$

$-22 + 71i$

EXAMPLE 4 **Multiply complex numbers**

Write the expression as a complex number in standard form.

$(3 + i)(5 - i)$

$15 - 3i + 5i - i^2$

$15 + 2i - (-1)$

$15 + 2i + 1$

$16 + 2i$

$i(9 - i)$

$9i - i^2$

$9i - (-1)$

$9i + 1$

$1 + 9i$

EXAMPLE 5 **Divide complex numbers**

Write the quotient $\frac{7+5i}{1-4i}$ in standard form.

$$\frac{(7+5i)}{(1-4i)}\cdot\frac{(1+4i)}{(1+4i)} = \frac{7+28i+5i+20i^2}{(1)^2-(4i)^2}$$

$$\frac{7+33i+20(-1)}{1-(-16)} = \frac{-13+33i}{17} = \boxed{\frac{-13}{17}+\frac{33}{17}i}$$

EXAMPLE 5 **Divide complex numbers**

Write the expression as a complex number in standard form.

$$\frac{5}{1+i}$$

$$\frac{5}{1+i}\cdot\frac{1-i}{1-i}$$

$$\frac{5-5i}{(1)^2-(i)^2}=\frac{5-5i}{1+1}=\frac{5-5i}{2}$$

$$\frac{5}{2}-\frac{5}{2}i$$

$$\frac{5+2i}{3-2i}\cdot\frac{3+2i}{3+2i}$$

$$\frac{15+10i+6i+4i^2}{(3)^2-(2i)^2}$$

$$\frac{11+16i}{9--4}$$

$$\frac{11}{13}+\frac{16}{13}i$$

EXAMPLE 6 Plot complex numbers

Plot the complex numbers in the same complex plane.

(3,-2)

a. $3 - 2i$

(-2,4)

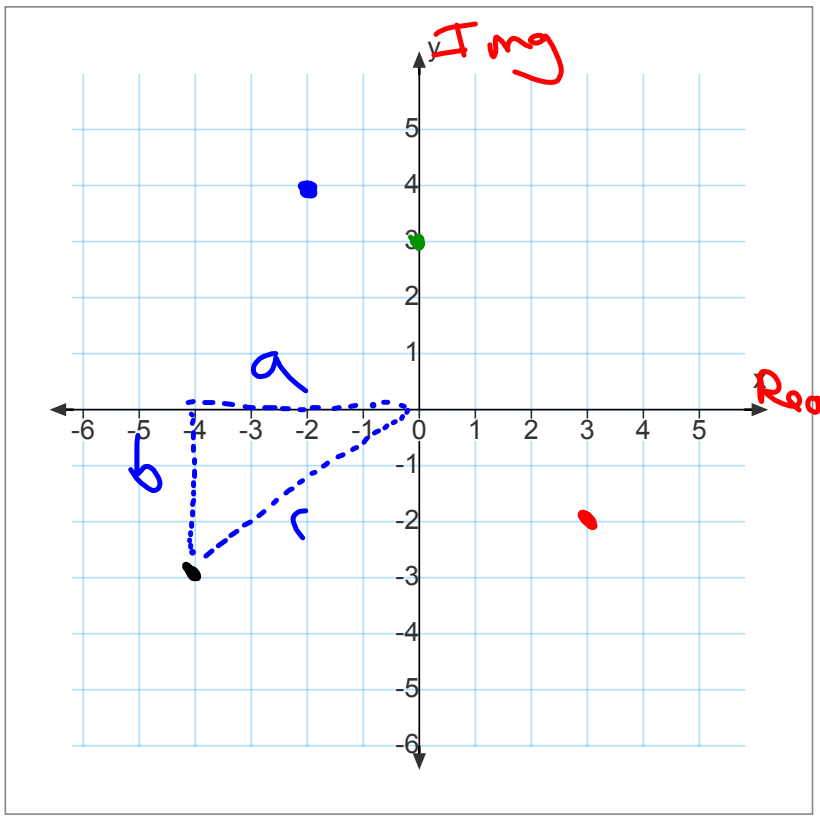
b. $-2 + 4i$

(0,3)

c. $3i$

d. $-4 - 3i$

(-4,-3)



EXAMPLE 7 **Find absolute values of complex numbers**

Find the absolute value of (a) $-4 + 3i$ and (b) $-3i$.

a) $|-4+3i| = \sqrt{(-4)^2+(3)^2}$

$= \sqrt{16+9} = \sqrt{25} = \boxed{5}$

b) $|-3i| = \sqrt{0^2+(-3)^2} = \sqrt{9} = \boxed{3}$