# 4.7 Complete the Square

**EXAMPLE 1** Solve a quadratic equation by finding square roots

Solve the equation by finding square roots.

$x^2 - 8x + 16 = 25$

$(x-4)(x-4) = 25$

$\sqrt{(x-4)^2} = \sqrt{25}$

$x - 4 = \pm 5$

$x = 4 \pm 5$

$x = 4+5$ $x = 4-5$

$x = 9$ $x = -1$

$x^2 + 6x + 9 = 36$

$3 \quad 3$

$\sqrt{(x+3)^2} = \sqrt{36}$

$x + 3 = \pm 6$

$x = -3 \pm 6$

$x = 3$ $x = -9$

**EXAMPLE 2** **Make a perfect square trinomial**

Find the value of $c$ that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

$$x^2 + 16x + c$$

$\left(\frac{16}{2}\right)^2 = (8)^2 = 64$

$c = 64$

$x^2 + 16x + 64$

$(x+8)^2$

**EXAMPLE 2** **Make a perfect square trinomial**

**Find the value of $c$ that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.**

$x^2 + 14x + c$

$\left(\frac{14}{2}\right)^2 = (7)^2 = 49$

$c = 49$

$(x+7)^2$

$x^2 + 22x + c$

$\left(\frac{22}{2}\right)^2 = (11)^2$

$c = 121$

$(x+11)^2$

$x^2 - 9x + c$

$\left(\frac{-9}{2}\right)^2$

$\frac{-9}{2} \cdot \frac{-9}{2} = \frac{81}{4}$

$c = \frac{81}{4}$

$\left(x - \frac{9}{2}\right)^2$

**EXAMPLE 3** **Solve $ax^2 + bx + c = 0$ when $a = 1$**

Solve $x^2 - 12x + 4 = 0$ by completing the square.

$$x^2 - 12x + 4 = 0$$
$$-4 \quad -4$$
$$x^2 - 12x + \boxed{36} = -4 + \boxed{36}$$
$$\left(\frac{-12}{2}\right)^2 = (-6)^2$$
$$\sqrt{(x-6)^2} = \sqrt{32}$$
$$\sqrt{16}\sqrt{2}$$
$$x - 6 = \pm 4\sqrt{2}$$
$$\boxed{x = 6 \pm 4\sqrt{2}}$$

**EXAMPLE 3** **Solve $ax^2 + bx + c = 0$ when $a = 1$**

**Solve the equation by completing the square.**

$$x^2 + 6x + 4 = 0$$

$$x^2 + 6x + \frac{9}{} = -4 + \frac{9}{}$$

$$\left(\frac{6}{2}\right)^2 \rightarrow (3)^2$$

$$\sqrt{(x+3)^2} = \sqrt{5}$$

$$x + 3 = \pm\sqrt{5}$$

$$\boxed{x = -3 \pm \sqrt{5}}$$

**EXAMPLE 4** Solve $ax^2 + bx + c = 0$ when $a \neq 1$

**Solve the equation by completing the square.**

$$2n^2 - 4n - 14 = 0$$

$$\frac{2n^2}{2} - \frac{4n}{2} - \frac{14}{2} = \frac{0}{2}$$

$$n^2 - 2n - 7 = 0$$

$$n^2 - 2n + \frac{1}{} = 7 + \frac{1}{}$$

$$\left(\frac{-2}{2}\right)^2 = (-1)^2$$

$$\sqrt{(n-1)^2} = \sqrt{8}$$

$$n - 1 = \pm 2\sqrt{2}$$

$$\boxed{n = 1 \pm 2\sqrt{2}}$$

**EXAMPLE 4** **Solve $ax^2 + bx + c = 0$ when $a \neq 1$**

**Solve the equation by completing the square.**

$$6x(x + 8) = 12$$

$$\frac{6x^2}{6} + \frac{48x}{6} = \frac{12}{6}$$

$$x^2 + 8x + 16 = 2 + 16$$

$$\left(\frac{8}{2}\right)^2 = (4)^2$$

$$\sqrt{(x+4)^2} = \sqrt{18}$$

$$x + 4 = \pm 3\sqrt{2}$$

$$\boxed{x = -4 \pm 3\sqrt{2}}$$