12.1 Define and Use Sequences and Series

Series: a summation of a seq.

seq
2, 4, 6, 8, .
1, 3, 9, 27

series
2+4+6+8+ ...
1+3+9+27

"sum of all"

$$\sum_{i=1}^{n} a_i$$

end
rule
start

EXAMPLE 4 **Write series using summation notation**

Write the series using summation notation.

a. $25 + 50 + 75 + \cdots + 250$

1 2 3 10

+25 +25

$a_n = 25n$

$= 25 + (n-1)(25)$

$\sum_{i=1}^{10} 25_i$

b. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \cdots$

1 2 3 4

$\sum_{i=1}^{\infty} \frac{n}{n+1}$

EXAMPLE 4 **Write series using summation notation**

Write the series using summation notation.

$\frac{1}{2} + \frac{4}{5} + \frac{9}{10} + \frac{16}{17} + \cdots$

1 2 3 4

$\sum_{i=1}^{\infty} \frac{n^2}{n^2+1}$

$5 + 10 + 15 + \cdots + 100$

1 20

5 5

$5+(n-1)(5) = S_n$

$\sum_{i=1}^{20} S_n$

EXAMPLE 5 Find the sum of a series

Find the sum of the series.

$$\sum_{k=4}^{8} (3 + k^2)$$

$(3+4^2)+(3+5^2)+(3+6^2)+$
$(3+7^2)+(3+8^2)$

$=19+28+39+52+67$

$=205$

$$\sum_{i=1}^{5} 8i = 8\sum_{i=1}^{5} i$$

$8(1)+8(2)+8(3)+8(4)+8(5)$

$8+16+24+32+40$

40

40

120

$8(1+2+3+4+5)$

$8(15)=120$

EXAMPLE 5 Find the sum of a series

Find the sum of the series.

$$\sum_{k=3}^{7} (k^2 - 1)$$

$(3^2-1)+(4^2-1)+(5^2-1)+(6^2-1)$
$+(7^2-1)$

$=8+15+24+35+48$

32 50 80 130

$$\sum_{i=1}^{34} 1$$

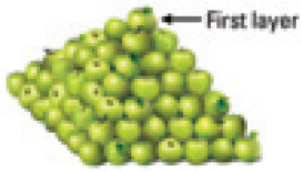
$1+1+\ldots+1$

34

34

EXAMPLE 6 Use a formula for a sum

RETAIL DISPLAYS How many apples are in the stack in Example 3 on page 795?



Sum of first n positive integers	Sum of squares of first n positive integers
$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$a_n = n^2$

$1+4+9+\ldots+49$

$$\sum_{i=1}^{7} i^2 = \frac{7(7+1)(2(7)+1)}{6} = \frac{7 \cdot 8 \cdot 15}{6} = 140$$

EXAMPLE 6 Use a formula for a sum

Find the sum of the series.

Sum of first n positive integers	Sum of squares of first n positive integers
$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$$\sum_{i=1}^{100} i$$

$1+2+3+4+\ldots+100$

$\frac{100(100+1)}{2}$

$50(101) = 5050$

$1+4+9+\ldots+2500$

$1 \quad 2 \quad 3 \quad \ldots \quad 50$

$\sum_{i=1}^{50} i^2 = \frac{50(50+1)(2(50)+1)}{6}$

$= \frac{50(51)(101)}{6}$

$= 42{,}925$

12.2 Analyze Arithmetic Sequences and Series

EXAMPLE 5

What is the sum of the arithmetic series $\sum_{i=1}^{20}(3+5i)$?

$(3+5(1))+(3+5(2))+(3+5(3))+\ldots+(3+5(20))$

$8+13+18+\ldots+103=1110$

$S_n=\frac{n}{2}(a_1+a_n)$

EXAMPLE 5

Find the sum of the arithmetic series $\sum_{i=1}^{12}(2+7i)$

$\frac{n}{2}(a_1+a_n)$

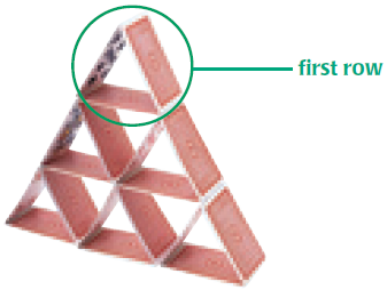
$\frac{12}{2}(9+86)$

$6(95)$

570

EXAMPLE 6 Use an arithmetic sequence and series in real life

HOUSE OF CARDS You are making a house of cards similar to the one shown.



a. Write a rule for the number of cards in the nth row if the top row is row 1.

b. What is the total number of cards if the house of cards has 14 rows?

$a_1 = 3$

$a_2 = 6$

$a_3 = 9$

$a_n = 3 + (n-1)(3)$

$$\sum_{i=1}^{14} 3_n = \frac{14}{2}(3+42)$$

$$= 7(45)$$

$$= 315 \text{ cards}$$

FINDING SUMS Find the sum of the arithmetic series.

46. $2 + 6 + 10 + \cdots + 58$

4 4

$a_n = 2 + (n-1)(4)$

$58 = 2 + (n-1)(4)$

$\frac{56}{4} = \frac{(n-1)(4)}{4}$

$14 = n - 1$

$15 = n$

$\frac{n}{2}(a_1 + a_n)$

$\frac{15}{2}(2 + 58)$

$\frac{15(60)}{2}$

$15(30)$

450

12.3 Analyze Geometric Sequences and Series

EXAMPLE 5 Find the sum of a geometric series

Find the sum of the geometric series $\sum_{i=1}^{16} 4(3)^{i-1}$.

$4 + 12 + 36 + \ldots +$ ____

$\cdot 3 \quad \cdot 3$

$r = 3$

$a_1 = 4$

$a_1 (r)^{n-1}$

$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$

$= 4\left(\frac{1-3^{16}}{1-3}\right) = 4\left(\frac{-43046720}{-2}\right)$

$= 86093440$

EXAMPLE 5 **Find the sum of a geometric series**

Find the sum of the geometric series $\sum_{i=1}^{8} 6(-2)^{i-1}$.

$a_1\left(\frac{1-r^n}{1-r}\right)$

$a_1(r)^{n-1}$

$6\left(\frac{1-(-2)^8}{1-(-2)}\right)$

$\cancel{6}^{2}\left(\frac{1-256}{\cancel{3}}\right) = 2(-255) = \boxed{-510}$

FINDING SUMS Find the sum of the geometric series.

49. $\sum_{i=1}^{8} 6(4)^{i-1}$

$a_1 = 6$

$r = 4$

$n = 8$

$$a_1\left(\frac{1-r^n}{1-r}\right) = 6\left(\frac{1-4^8}{1-4}\right)$$

$$= 6\left(\frac{1-65536}{-3}\right)$$

$$= 2(65535)$$

$$= 131{,}070$$