1 PLAN AND PREPARE

Warm-Up Exercises

Evaluate each expression.

1. $3x - 1$ for $x = -2$ **−7**

2. $6 + 5x$ for $x = 4$ **26**

3. $-2x + 9$ for $x = -1$ **11**

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 3, p. 71

What is a step function? **Tell students they will learn how to answer this question by learning about piecewise functions.**

3 TEACH

Extra Example 1

Evaluate the function

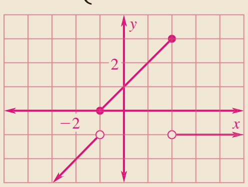
$$g(x) = \begin{cases} 4x - 3, & \text{if } x > 3 \\ 5x + 2, & \text{if } x \le 3 \end{cases}$$

when $x = -2$ and $x = 5$. **−8, 17**

Extra Example 2

Graph the function

$$f(x) = \begin{cases} x, & \text{if } x < -1 \\ x + 1, & \text{if } -1 \le x \le 2 \\ -1, & \text{if } x > 2 \end{cases}.$$



NCTM STANDARDS

Standard 2: Understand functions; Represent situations using algebraic symbols

Extension

Use after Lesson 2.7

Use Piecewise Functions

GOAL *Evaluate, graph, and write piecewise functions.*

Key Vocabulary
- **piecewise function**
- **step function**

A **piecewise function** is defined by at least two equations, each of which applies to a different part of the function's domain. One example of a piecewise function is the absolute value function $f(x) = |x|$, which can be defined by the equations $y = -x$ for $x < 0$ and $y = x$ for $x \ge 0$. Another example is given below.

$$g(x) = \begin{cases} 2x - 1, & \text{if } x \le 1 \\ 3x + 1, & \text{if } x > 1 \end{cases}$$

The equation $y = 2x - 1$ gives the value of $g(x)$ when x is less than or equal to 1, and the equation $y = 3x + 1$ gives the value of $g(x)$ when x is greater than 1.

EXAMPLE 1 Evaluate a piecewise function

Evaluate the function $g(x)$ above when (a) $x = 1$ and (b) $x = 5$.

Solution

a. $g(x) = 2x - 1$ — **Because $1 \le 1$, use first equation.**

$g(1) = 2(1) - 1 = 1$ — **Substitute 1 for x and simplify.**

b. $g(x) = 3x + 1$ — **Because $5 > 1$, use second equation.**

$g(5) = 3(5) + 1 = 16$ — **Substitute 5 for x and simplify.**

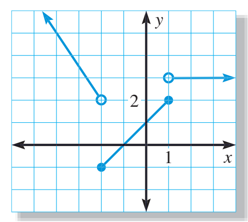
EXAMPLE 2 Graph a piecewise function

Graph the function $$f(x) = \begin{cases} -\frac{3}{2}x - 1, & \text{if } x < -2 \\ x + 1, & \text{if } -2 \le x \le 1 \\ 3, & \text{if } x > 1 \end{cases}$$

Solution

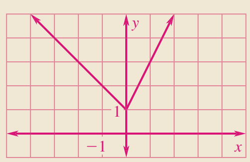
STEP 1 To the left of $x = -2$, graph $y = -\frac{3}{2}x - 1$. Use an open dot at $(-2, 2)$ because the equation $y = -\frac{3}{2}x - 1$ does not apply when $x = -2$.

STEP 2 From $x = -2$ to $x = 1$, inclusive, graph $y = x + 1$. Use solid dots at $(-2, -1)$ and $(1, 2)$ because the equation $y = x + 1$ applies to both $x = -2$ and $x = 1$.



STEP 3 To the right of $x = 1$, graph $y = 3$. Use an open dot at $(1, 3)$ because the equation $y = 3$ does not apply when $x = 1$.

5.



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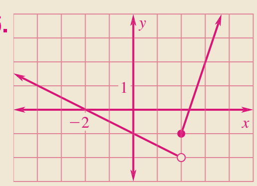

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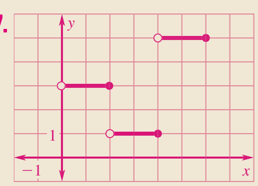
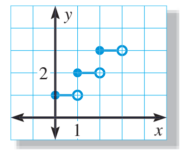


EXAMPLE 3 Write a piecewise function

Write a piecewise function for the graph shown.



Solution

For x between 0 and 1, including $x = 0$, the graph is the line segment given by $y = 1$.

For x between 1 and 2, including $x = 1$, the graph is the line segment given by $y = 2$.

For x between 2 and 3, including $x = 2$, the graph is the line segment given by $y = 3$. So, a piecewise function for the graph is as follows:

$$f(x) = \begin{cases} 1, & \text{if } 0 \le x < 1 \\ 2, & \text{if } 1 \le x < 2 \\ 3, & \text{if } 2 \le x < 3 \end{cases}$$

STEP FUNCTIONS The piecewise function in Example 3 is called a **step function** because its graph resembles a set of stairs. A step function is defined by a constant value over each part of its domain. The constant values can increase with each "step" as in Example 3, or they can decrease with each step.

PRACTICE

EXAMPLE 1 on p. 130 for Exs. 1–4

EVALUATING FUNCTIONS **Evaluate the function below for the given value of x.**

$$f(x) = \begin{cases} 9x - 4, & \text{if } x > 3 \\ \frac{1}{2}x + 1, & \text{if } x \le 3 \end{cases}$$

1. $f(-4)$ **−1** **2.** $f(2)$ **2** **3.** $f(3)$ $\frac{5}{2}$ **4.** $f(5)$ **41**

EXAMPLE 2 on p. 130 for Exs. 5–8

GRAPHING FUNCTIONS **Graph the function.** **5–7. See margin.**

5. $f(x) = \begin{cases} 2x + 1, & \text{if } x \ge 0 \\ -x + 1, & \text{if } x < 0 \end{cases}$

6. $g(x) = \begin{cases} -\frac{1}{2}x - 1, & \text{if } x < 2 \\ 3x - 7, & \text{if } x \ge 2 \end{cases}$

7. $h(x) = \begin{cases} 3, & \text{if } 0 < x \le 2 \\ 1, & \text{if } 2 < x \le 4 \\ 5, & \text{if } 4 < x \le 6 \end{cases}$

8. POSTAL RATES In 2005, the cost C (in dollars) to send U.S. Postal Service Express Mail up to 5 pounds depended on the weight w (in ounces) according to the function at the right.

$$C(w) = \begin{cases} 13.65, & \text{if } 0 < w \le 8 \\ 17.85, & \text{if } 8 < w \le 32 \\ 21.05, & \text{if } 32 < w \le 48 \\ 24.20, & \text{if } 48 < w \le 64 \\ 27.30, & \text{if } 64 < w \le 80 \end{cases}$$

a. Graph the function. **See margin.**

b. What is the cost to send a parcel weighing 2 pounds 9 ounces using Express Mail? **$21.05**

EXAMPLE 3 on p. 131 for Exs. 9–10

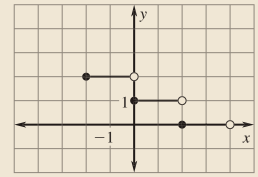
SPECIAL STEP FUNCTIONS **Write and graph the piecewise function described using the domain $-3 \le x \le 3$.** **9–10. See margin.**

9. Rounding Function The output $f(x)$ is the input x rounded to the nearest integer. (If the decimal part of x is 0.5, then x is rounded up.)

10. Greatest Integer Function The output $f(x)$ is the greatest integer less than or equal to the input x.

Extra Example 3

Write a piecewise function for the graph shown.



$$f(x) = \begin{cases} 2, & \text{if } -2 \le x < 0 \\ 1, & \text{if } 0 \le x < 2 \\ 0, & \text{if } 2 \le x < 4 \end{cases}$$

Key Questions to Ask for Example 3

- What is the domain of $f(x)$? $0 \le x < 3$
- What is the range of $f(x)$? **1, 2, 3**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: What is a step function?

- **A piecewise function is defined by at least two equations, each of which applies to a different part of the function's domain.**
- **To graph a piecewise function, graph each part of the function for its given domain.**

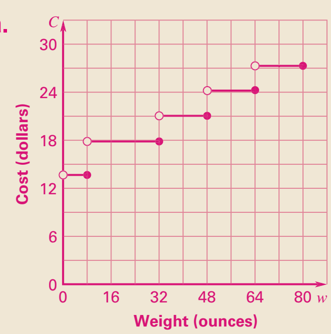
A step function is defined by a constant value over each part of its domain.

4 PRACTICE AND APPLY

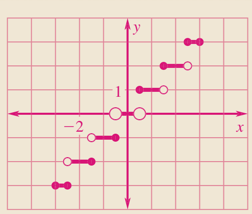
Avoiding Common Errors

Remind students that because piecewise functions are in fact functions, their graphs should pass the vertical line test. Performing this test may help them avoid including a point in two or more pieces of the graph of the function.

8a.



9.



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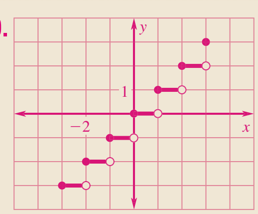

2 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

- relation, *p. 72*
- domain, range, *p. 72*
- function, *p. 73*
- equation in two variables, *p. 74*
- solution, graph of an equation in two variables, *p. 74*
- independent variable, *p. 74*
- dependent variable, *p. 74*
- linear function, *p. 75*
- function notation, *p. 75*
- slope, *p. 82*
- parallel, perpendicular, *p. 84*
- rate of change, *p. 85*
- parent function, *p. 89*
- *y*-intercept, *p. 89*
- slope-intercept form, *p. 90*
- *x*-intercept, *p. 91*
- standard form of a linear equation, *p. 91*
- point-slope form, *p. 98*
- direct variation, *p. 107*
- constant of variation, *p. 107*
- scatter plot, *p. 113*
- positive correlation, *p. 113*
- negative correlation, *p. 113*
- correlation coefficient, *p. 114*
- best-fitting line, *p. 114*
- absolute value function, *p. 123*
- vertex of an absolute value graph, *p. 123*
- transformation, *p. 123*
- translation, *p. 123*
- reflection, *p. 124*
- linear inequality in two variables, *p. 132*
- solution, graph of a linear inequality in two variables, *p. 132*
- half-plane, *p. 132*

VOCABULARY EXERCISES

1. Copy and complete: The linear equation $5x - 4y = 16$ is written in _?_ form. **standard**

2. Copy and complete: A set of data pairs (x, y) shows a _?_ correlation if y tends to decrease as x increases. **negative**

3. Copy and complete: Two variables x and y show _?_ if $y = ax$ and $a \neq 0$. **direct variation**

4. WRITING *Explain* what distinguishes a function from a relation.
Sample answer: **A function has exactly one output for each input, while a relation can have more than one output for each input.**

REVIEW EXAMPLES AND EXERCISES

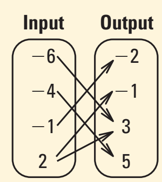
Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 2.

2.1 Represent Relations and Functions *pp. 72–79*

EXAMPLE

Tell whether the relation given by the ordered pairs (−6, 3), (−4, 5), (−1, −2), (2, −1), and (2, 3) is a function.

The relation is *not* a function because the input 2 is mapped onto both −1 and 3, as shown in the mapping diagram.



EXERCISES

Consider the relation given by the ordered pairs. Identify the domain and range. Then tell whether the relation is a function.

5. (−2, −2), (−1, 0), (2, 6), (3, 8)

6. (−1, −5), (1, 2), (3, 4), (1, −7)

7. Tell whether $f(x) = 16 - 7x$ is a linear function. Then find $f(-5)$. **linear function; 51**

5. domain: −2, −1, 2, 3, range: −2, 0, 6, 8; function

6. domain: −1, 1, 3, range: −7, −5, 2, 4; not a function

EXAMPLES 1, 2, and 5 on pp. 72–75 for Exs. 5–7

Extra Example 2.1

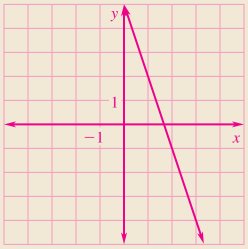
Tell whether the relation given by the ordered pairs (−2, 3), (5, 0), (3, −1), (−4, −4), and (7, 13) is a function. **yes**

Extra Example 2.2

Find the slope of the line passing through (8, 2) and (−5, 1). $\frac{1}{13}$

Extra Example 2.3

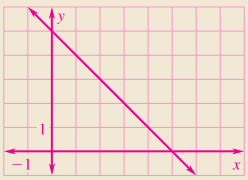
Graph $3x + y = 5$.



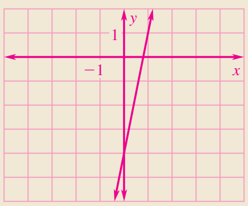
Extra Example 2.4

Write an equation of the line that passes through (−1, 6) and (3, −2). $y = -2x + 4$

12.



13.



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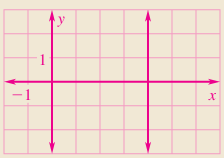

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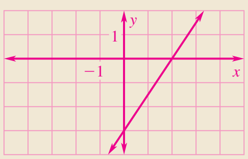


2 CHAPTER REVIEW

2.2 Find Slope and Rate of Change *pp. 82–88*

EXAMPLE

Find the slope m of the line passing through the points (−4, 12) and (3, −2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 12}{3 - (-4)} = \frac{-14}{7} = -2$$

EXERCISES

EXAMPLE 2 on p. 82 for Exs. 8–11

Find the slope of the line passing through the given points.

8. (−2, −1), (4, 3) $\frac{2}{3}$ **9.** (1, −5), (1, 2) undefined **10.** (5, −3), (1, 7) $-\frac{5}{2}$ **11.** (6, 2), (−8, 2) 0

2.3 Graph Equations of Lines *pp. 89–96*

EXAMPLE

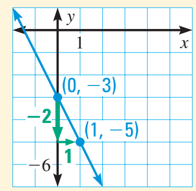
Graph $3 + y = -2x$.

STEP 1 Write the equation in slope-intercept form, $y = -2x - 3$.

STEP 2 The y-intercept is −3. So, plot the point (0, −3).

STEP 3 The slope is −2. Plot a second point by starting at (0, −3) and then moving down 2 units and right 1 unit.

STEP 4 Draw a line through the two points.



EXERCISES

EXAMPLES 1, 2, and 4 on pp. 89–92 for Exs. 12–15

Graph the equation. 12–15. See margin.

12. $y = 5 - x$ **13.** $y - 5x = -4$ **14.** $x = 4$ **15.** $6x - 4y = 12$

2.4 Write Equations of Lines *pp. 98–104*

EXAMPLE

Write an equation of the line that passes through (−2, 5) and (−4, −1).

The slope is $m = \frac{-1 - 5}{-4 - (-2)} = 3$. Use the point-slope form with $(x_1, y_1) = (-2, 5)$.

$y - y_1 = m(x - x_1)$	**Use point-slope form.**
$y - 5 = 3(x - (-2))$	**Substitute for m, x_1, and y_1.**
$y = 3x + 11$	**Write in slope-intercept form.**

EXERCISES

16. $y = -2x - 2$
17. $y = -\frac{3}{4}x + 2$
18. $y = -x - 3$

EXAMPLE 4 on p. 100 for Exs. 16–18

Write an equation of the line that passes through the given points.

16. (−3, 4), (2, −6) **17.** (−4, 5), (12, −7) **18.** (−4, 1), (3, −6)

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Chapter Review Practice

2.5 Model Direct Variation pp. 107–111

EXAMPLE

The variables x and y vary directly, and $y = 76$ when $x = -8$. Write an equation that relates x and y. Then find y when $x = -6$.

$y = ax$ — Write direct variation equation.

$76 = a(-8)$ — Substitute 76 for y and -8 for x.

$-9.5 = a$ — Solve for a.

An equation that relates x and y is $y = -9.5x$. When $x = -6$, $y = -9.5(-6) = 57$.

EXERCISES

EXAMPLE 2 on p. 108 for Exs. 19–22

The variables x and y vary directly. Write an equation that relates x and y. Then find y when $x = 3$.

19. $x = 6, y = -48$ — $y = -8x; -24$

20. $x = -9, y = 15$ — $y = -\frac{5}{3}x; -5$

21. $x = -3, y = 2.4$ — $y = -0.8x; -2.4$

22. PHYSICS Charles's Law states that when pressure is constant, the volume V of a gas varies directly with its temperature T (in kelvins). A gas occupies 4.8 liters at a temperature of 300 kelvins. Write an equation that gives V as a function of T. What is the volume of the gas when the temperature is 420 kelvins? $V = 0.016T$; 6.72 L

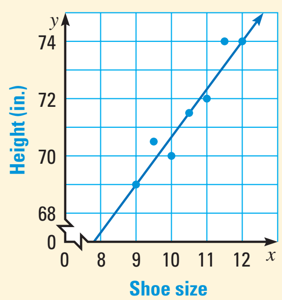
2.6 Draw Scatter Plots and Best-Fitting Lines pp. 113–120

EXAMPLE

The table shows the shoe size x and height y (in inches) for 7 men. Approximate the best-fitting line for the data.

x	9	9.5	10	10.5	11	11.5	12
y	69	70.5	70	71.5	72	74	74

Draw a scatter plot and sketch the line that appears to best fit the data points.



Choose two points on the line, such as (9, 69) and (12, 74). Use the points to find an equation of the line.

The slope is $m = \frac{74 - 69}{12 - 9} = \frac{5}{3} \approx 1.67$.

An equation is $y - 69 = 1.67(x - 9)$, or $y = 1.67x + 54$.

EXERCISES

EXAMPLE 3 on p. 115 for Ex. 23

Approximate the best-fitting line for the data.

23.

x	−2	−1	0	1	2	3	4	5
y	4	3	2.5	2	0.5	−1	−2	−3

Sample answer: $y = -x + 2.3$

Extra Example 2.5

The variables x and y vary directly, and $y = -77$ when $x = -5$. Write and equation that relates x and y. Then find y when $x = 12$. $y = 15.4x$; 184.8

Extra Example 2.6

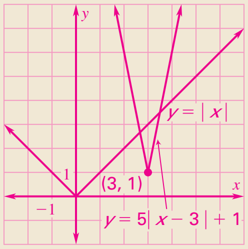
The table shows the shoe size x and height y (in inches) for 6 women. Approximate the best-fitting line for the data. $y = 3.3x + 36$

x	6	8.5	8	8.5	9	9.5
y	57	60	62	65	66	70

2 CHAPTER REVIEW

Extra Example 2.7

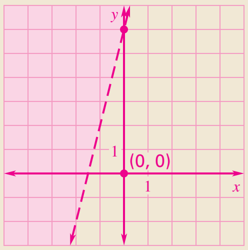
Graph $y = 5|x - 3| + 1$. Compare the graph with the graph of $y = |x|$.



The graph of $y = |5x - 3| + 1$ is the graph of $y = |x|$ first vertically stretched by a factor of 5, then translated right 3 units and up 1 unit.

Extra Example 2.8

Graph $-4x + y > 6$ in a coordinate plane.



2.7 Use Absolute Value Functions and Transformations *pp. 123–129*

EXAMPLE

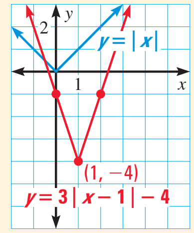
Graph $y = 3|x - 1| - 4$. Compare the graph with the graph of $y = |x|$.

STEP 1 **Identify** and plot the vertex, $(h, k) = (1, -4)$.

STEP 2 **Plot** another point on the graph, such as $(0, -1)$. Use symmetry to plot a third point, $(2, -1)$.

STEP 3 **Connect** the points with a V-shaped graph.

STEP 4 **Compare** with $y = |x|$. The graph of $y = 3|x - 1| - 4$ is the graph of $y = |x|$ stretched vertically by a factor of 3, then translated right 1 unit and down 4 units.



EXERCISES

EXAMPLES 1, 2, 3, and 4 on pp. 123–125 for Exs. 24–27

Graph the function. *Compare* the graph to the graph of $y = |x|$. 24–26. See margin.

24. $y = |x - 3| + 2$ **25.** $y = \frac{3}{4}|x|$ **26.** $f(x) = -4|x + 2| + 3$

27. FINANCE Analysts predict that a company will report earnings of \$1.50 per share in the next quarter. The function $d = |a - 1.50|$ gives the absolute difference d between the actual earnings a and the predicted earnings. Graph the function. For what value(s) of a will d be \$.25? See margin for art; \$1.75, \$1.25.

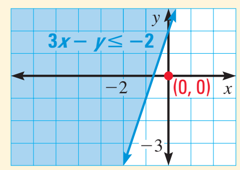
2.8 Graph Linear Inequalities in Two Variables *pp. 132–138*

EXAMPLE

Graph $3x - y \le -2$ in a coordinate plane.

STEP 1 **Graph** the boundary line $3x - y = -2$. Use a solid line because the inequality symbol is $\le$.

STEP 2 **Test** the point (0, 0). Because (0, 0) is *not* a solution of the inequality, shade the half-plane that does not contain (0, 0).



EXERCISES

EXAMPLES 2, 3, and 4 on pp. 132–134 for Exs. 28–34

Tell whether the given ordered pair is a solution of the inequality.

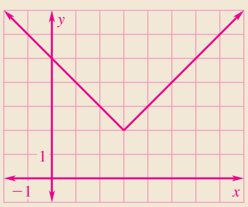
28. $-y \le 5x$; (0, 1) solution **29.** $y > -3x - 7$; (−4, 6) solution **30.** $3x - 4y < -8$; (−2, 0) not a solution

Graph the inequality in a coordinate plane. 31–33. See margin.

31. $-4y < 16$ **32.** $y - 2x > 8$ **33.** $12x - 8y \le 24$

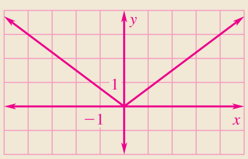
34. WIND ENERGY An electric company buys energy from "windmill farms" that have windmills of two sizes, one producing 1.5 megawatts of power and one producing 2.5 megawatts of power. The company wants a total power supply of at least 180 megawatts. Write and graph an inequality describing how many of each size of windmill it takes to supply the electric company. See margin.

24.



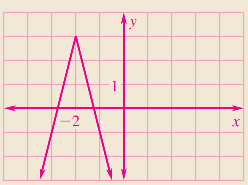
translated right 3 units and up 2 units

25.



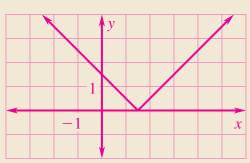
shrunk vertically by a factor of $\frac{3}{4}$

26.



reflected over the x-axis, stretched vertically by a factor of 4, translated left 2 units and up 3 units

27.



31–34. See Additional Answers beginning on p. AA1.