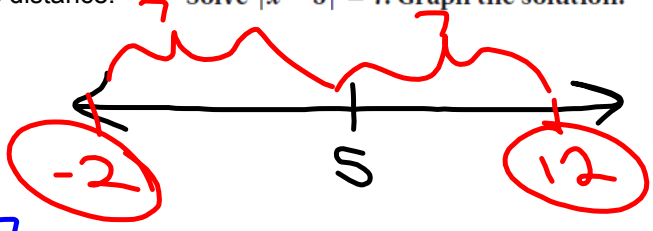
1.7 Solve Absolute Value Equations and Inequalities

EXAMPLE 1 Solve a simple absolute value equation

Absolute Value as distance: Solve $|x - 5| = 7$. Graph the solution.



$|-2-5|=7$

$|-7|=7$

$7=7$

$|12-5|=7$

$|7|=7$

$7=7$

EXAMPLE 2 **Solve an absolute value equation**

Solve $|5x - 10| = 45$.

$5x - 10 = 45$ $\quad +10 \quad +10$

$\frac{5x}{5} = \frac{55}{5}$

$x = 11$

$5x - 10 = -45$ $\quad +10 \quad +10$

$\frac{5x}{5} = \frac{-35}{5}$

$x = -7$

EXAMPLE 3 **Check for extraneous solutions**

Solve $|2x + 12| = 4x$. Check for extraneous solutions.

$2x+12=4x$

$12=2x$

$6=x$

$|2(6)+12|=4(6)$

$|24|=24$

$24=24$

$2x+12=-4x$

$12=-6x$

$-2=x$

$|2(-2)+12|=4(-2)$

$|8|=-8$

$8=-8$

Solve the equation. Check for extraneous solutions.

$$|4x - 1| = 2x + 9$$

$4x-1=2x+9$

$2x-1=9$

$2x=10$

$x=5$

$19=19$

$4x-1=-(2x+9)$

$4x-1=-2x-9$

$6x-1=-9$

$6x=-8$

$x=\frac{-8}{6}=\frac{-4}{3}$

$\left|4\left(\frac{-4}{3}\right)-1\right|=2\left(\frac{-4}{3}\right)+9$

$\left|-\frac{16}{3}-\frac{3}{3}\right|=-\frac{8}{3}+\frac{27}{3}$

$\left|-\frac{19}{3}\right|=\frac{19}{3}$

$\frac{19}{3}=\frac{19}{3}$

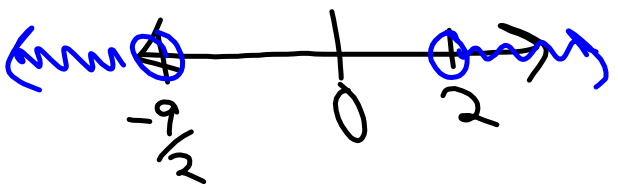
EXAMPLE 4 **Solve an inequality of the form $|ax + b| > c$**

Solve $|4x + 5| > 13$. Then graph the solution.

$4x+5>13$ $\quad$ $4x+5<-13$

$4x>8$ $\quad$ $4x<-18$

$x>2$ $\quad$ $x<\frac{-18}{4}=-\frac{9}{2}$



Solve the inequality. Then graph the solution.

$$|7 - x| \leq 4$$

$7 - x \leq 4$ $\quad$ $7 - x \geq -4$

$-7 \quad -7$ $\quad$ $-7 \quad -7$

$\frac{-x}{-1} \leq \frac{-3}{-1}$ $\quad$ $\frac{-x}{-1} \geq \frac{-11}{-1}$

$x \geq 3$ $\quad$ $x \leq 11$