

12.1 Define and Use Sequences and Series

Warm-Up Exercises

Evaluate $x^2 + 3$ for $x = 1, 2, 3,$ and 4 .

$$\rightarrow (1)^2 + 3 = 4$$

$$\rightarrow (2)^2 + 3 = 7$$

$$\rightarrow (3)^2 + 3 = 12$$

$$\rightarrow (4)^2 + 3 = 19$$

4, 7, 12, 19, ...

$n=1$ 2 3 4 5

Describe the pattern and find the next 2 terms:

-2, 1, 4, ... 7 10
 \downarrow \downarrow \downarrow \downarrow
 $+3$ $+3$ $+3$ $+3$

$$a_n = 3n - 5$$

EXAMPLE 1 Write terms of sequences

Write the first six terms of the sequence.

$$a_n = 2n + 5$$

$$a_1 = 2(1) + 5 = 7$$

$$a_2 = 2(2) + 5 = 9$$

$$a_3 = 2(3) + 5 = 11$$

$$a_4 = \quad = 13$$

$$a_5 = \quad = 15$$

$$a_6 = \quad = 17$$

7, 9, 11, 13, 15, 17, ...

EXAMPLE 1 Write terms of sequences

Write the first six terms of the sequence.

$$f(n) = (-3)^{n-1}$$

$$a_1 = (-3)^{1-1} = (-3)^0 = 1$$

$$a_2 = (-3)^{2-1} = (-3)^1 = -3$$

$$a_3 = (-3)^{3-1} = (-3)^2 = 9$$

$$a_4 = \phantom{(-3)^{4-1}} = -27$$

$$a_5 = \phantom{(-3)^{5-1}} = 81$$

$$a_6 = (-3)^{6-1} = (-3)^5 = -243$$

1, -3, 9, -27, 81, -243, ...

$$a_n = \frac{n}{n+1}$$

$$a_1 = \frac{1}{1+1} = \frac{1}{2}$$

$$a_2 = \frac{2}{2+1} = \frac{2}{3}$$

$$a_3 = \frac{3}{3+1} = \frac{3}{4}$$

$$a_4 = \phantom{\frac{4}{4+1}} = \frac{4}{5}$$

$$a_5 = \phantom{\frac{5}{5+1}} = \frac{5}{6}$$

$$a_6 = \phantom{\frac{6}{6+1}} = \frac{6}{7}$$

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \dots$

EXAMPLE 2 Write rules for sequences

Describe the pattern, write the next term, and write a rule for the n th term of the sequence (a) $-1, -8, -27, -64, \dots$ and (b) $0, 2, 6, 12, \dots$.

$$\begin{aligned} \text{a) } & -1, -8, -27, -64, \dots \\ & -(1)^3, -(2)^3, -(3)^3, -(4)^3, \dots \end{aligned}$$

$$a_5 = -(5)^3 = -125$$

$$a_n = -n^3$$

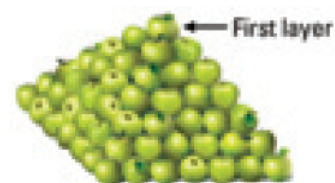
$$\begin{aligned} & 0, 2, 6, 12, \dots \\ & \quad \swarrow \quad \swarrow \quad \swarrow \\ & \quad +2 \quad +4 \quad +6 \\ & \quad \swarrow \quad \swarrow \\ & \quad +2 \quad +2 \end{aligned}$$

$$a_n = n(n-1) = n^2 - n$$

$$a_5 = 5(5-1) = 20$$

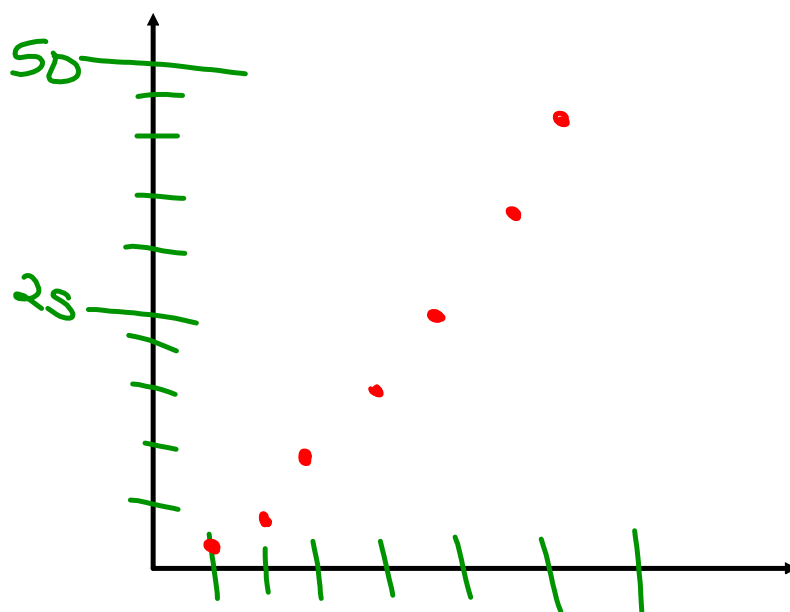
EXAMPLE 3 Solve a multi-step problem

RETAIL DISPLAYS You work in a grocery store and are stacking apples in the shape of a square pyramid with 7 layers. Write a rule for the number of apples in each layer. Then graph the sequence.



$$a_1 = 1 \quad \bullet$$
$$a_2 = 4 \quad \begin{matrix} \bullet \\ \bullet \\ \bullet \end{matrix}$$
$$a_3 = 9 \quad \begin{matrix} \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \end{matrix}$$

$$a_n = n^2$$



WRITING TERMS Write the first six terms of the sequence.

7. $a_n = 4^{n-1}$

$$a_1 = 4^{1-1} = 4^0 = 1$$

$$a_2 = 4^{2-1} = 4^1 = 4$$

$$a_3 = 4^{3-1} = 4^2 = 16$$

$$a_4 = \quad = 64$$

$$a_5 = \quad = 256$$

$$a_6 = \quad = 1024$$

1, 4, 16, 64, 256, 1024

WRITING RULES For the sequence, describe the pattern, write the next term, and write a rule for the n th term.

15. 1, 6, 11, 16, ... (21)

\checkmark \checkmark \checkmark \checkmark
 $+5$ $+5$ $+5$

$$a_n = 5n - 4$$

$$a_n = 5n$$

17. -4, 8, -12, 16, ...

$$a_n = (-1)^n (4n)$$

GRAPHING SEQUENCES Graph the sequence.

35. $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots, \frac{13}{2}$

