12.2 Analyze Arithmetic Sequences and Series

EXAMPLE 1 Identify arithmetic sequences

Tell whether the sequence is arithmetic.

a. $-4, 1, 6, 11, 16, \ldots$

$-4 \quad 1 \quad 6 \quad 11 \quad 16$

$+5 \quad +5 \quad +5 \quad +5$

Arithmetic

$[d = 5]$

b. $3, 5, 9, 15, 23, \ldots$

$3 \quad 5 \quad 9 \quad 15 \quad 23$

$+2 \quad +4 \quad +6 \quad +8$

Not arithmetic

EXAMPLE 2 **Write a rule for the *n*th term**

Write a rule for the nth term of the sequence. Then find a_{15}.

a. 4, 9, 14, 19, . . .

4 9 14 19

+5 +5 +5

$a_n = a_1 + (n-1)d$

$a_n = 4 + (n-1)(5)$

$a_{15} = 4 + (15-1)(5)$

$= 4 + (14)(5) = 74$

b. 60, 52, 44, 36, . . .

60 52 44 36

-8 -8 -8

$a_n = 60 + (n-1)(-8)$

$a_{15} = 60 + (15-1)(-8)$

$= 60 + (14)(-8)$

$= 60 - 112$

$= -52$

EXAMPLE 3 **Write a rule given a term and common difference**

One term of an arithmetic sequence is $a_{19} = 48$. The common difference is $d = 3$.

a. Write a rule for the nth term.

$a_{19} = 48 \quad d = 3$

$a_n = a_1 + (n-1)d$

$48 = a_1 + (19-1)(3)$

$48 = a_1 + (18)(3)$

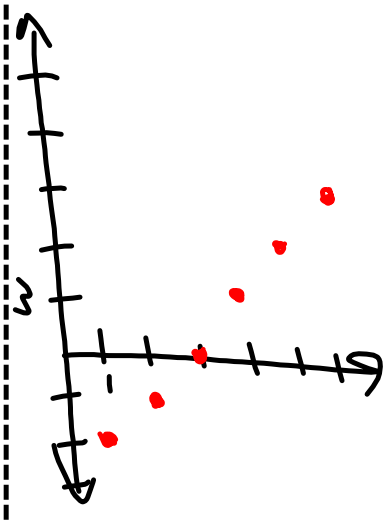
$48 = a_1 + 54$

$-6 = a_1$

$a_n = -6 + (n-1)(3)$

b. Graph the sequence.

$-6, -3, 0, 3, 6, 9$



EXAMPLE 4 **Write a rule given two terms**

Two terms of an arithmetic sequence are $a_8 = 21$ and $a_{27} = 97$. Find a rule for the nth term.

$$d = \frac{97-21}{27-8} = \frac{76}{19} = 4$$

$a_n = a_1 + (n-1)(d)$

$21 = a_1 + (8-1)(4)$

$21 = a_1 + 28$

$-7 = a_1$

$a_n = -7 + (n-1)(4)$

EXAMPLE 4 **Write a rule given two terms**

Write a rule for the *n*th term of the arithmetic sequence. Then find a_{20}.

$a_7 = 26$ $a_{16} = 71$

$$d = \frac{71-26}{16-7} = \frac{45}{9} = 5$$

$d = 5$

$a_n = a_1 + (n-1)(d)$

$26 = a_1 + (7-1)(5)$

$26 = a_1 + 30$

$-4 = a_1$

$a_n = -4 + (n-1)(5)$

IDENTIFYING ARITHMETIC SEQUENCES Tell whether the sequence is arithmetic. *Explain* why or why not.

11. $-\frac{5}{2}, -1, \frac{1}{2}, 2, \frac{7}{2}, \ldots$

$-\frac{5}{2}$, $-\frac{2}{2}$, $\frac{1}{2}$, $\frac{4}{2}$, $\frac{7}{2}$

$+\frac{3}{2}$ $+\frac{3}{2}$ $+\frac{3}{2}$ $+\frac{3}{2}$

Arithmetic; it has a common diff. of $\frac{3}{2}$

WRITING RULES Write a rule for the nth term of the arithmetic sequence. Then find a_{20}.

19. $2, \frac{5}{3}, \frac{4}{3}, 1, \frac{2}{3}, \ldots$

$\frac{6}{3}, \frac{5}{3}, \frac{4}{3}, \frac{3}{3}, \frac{2}{3} \ldots$

$-\frac{1}{3}$ $-\frac{1}{3}$ $-\frac{1}{3}$ $-\frac{1}{3}$

$a_1 = 2 \quad d = -\frac{1}{3}$

$a_n = 2 + (n-1)\left(-\frac{1}{3}\right)$

$a_{20} = 2 + (20-1)\left(-\frac{1}{3}\right)$

$= 2 - \frac{19}{3}$

$= \frac{6-19}{3} = \boxed{-\frac{13}{3}}$

WRITING RULES Write a rule for the nth term of the arithmetic sequence. Then graph the first six terms of the sequence.

23. $a_{16} = 52, d = 5$

$a_n = a_1 + (n-1)(d)$

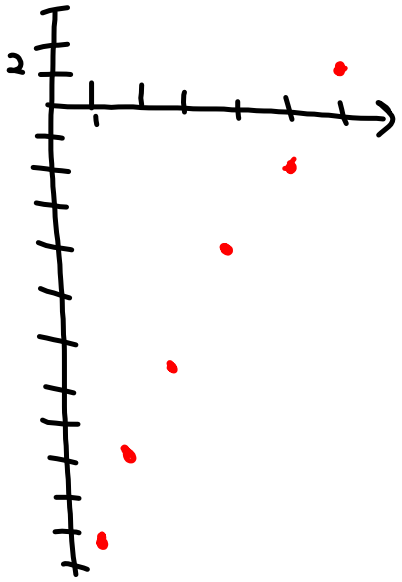
$52 = a_1 + (16-1)(5)$

$52 = a_1 + 75$

$-23 = a_1$

$a_n = -23 + (n-1)(5)$

$-23, -18, -13, -8, -3, 2$



WRITING RULES Write a rule for the nth term of the arithmetic sequence that has the two given terms.

36. $a_7 = 4, a_{12} = -9$

$d = \frac{-9-4}{12-7} = \frac{-13}{5}$

$a_n = a_1 + (n-1)(d)$

$4 = a_1 + (7-1)\left(\frac{-13}{5}\right)$

$4 = a_1 + \frac{-78}{5}$

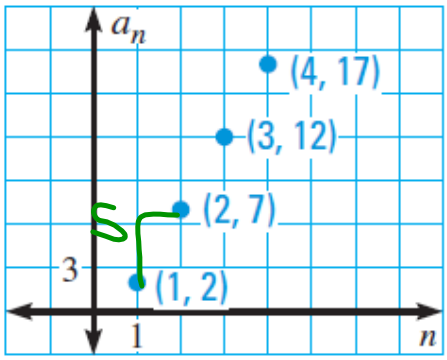
$\frac{20}{5} + \frac{78}{5} = a_1$

$a_1 = \frac{98}{5}$

$a_n = \frac{98}{5} + (n-1)\left(\frac{-13}{5}\right)$

USING GRAPHS Write a rule for the sequence whose graph is shown.

49.



$a_n = a_1 + (n-1)(d)$

$a_1 = 2$

$d = 5$

$a_n = 2 + (n-1)(5)$

STOP

Work on 12.2 pg 806 3-37odd

12.3 Analyze Geometric Sequences and Series

EXAMPLE 1 Identify geometric sequences

Tell whether the sequence is geometric.

4, 10, 18, 28, 40, . . .

$4r = 10 \quad r = \frac{10}{4} = \frac{5}{2}$

$10r = 18 \quad r = \frac{18}{10} = \frac{9}{5}$

Not Geo.

625, 125, 25, 5, 1, . . .

$r = \frac{125}{625} = \frac{1}{5}$

$r = \frac{25}{125} = \frac{1}{5}$

$r = \frac{5}{25} = \frac{1}{5}$

$r = \frac{1}{5}$

Yes

$-4, 8, -16, 32, -64, \ldots$

$r = \frac{8}{-4} = -2$

$r = \frac{-16}{8} = -2$

Yes

EXAMPLE 2 Write a rule for the *n*th term

Write a rule for the *n*th term of the sequence. Then find a_7.

a. 4, 20, 100, 500, . . .

$r = 5$

$a_1 = 4$

$a_n = a_1 \cdot r^{n-1}$

$a_n = 4(5)^{n-1}$

$a_7 = 4(5)^{7-1} = 4(5)^6$

$= 62500$

b. 152, −76, 38, −19, . . .

$r = \frac{-76}{152} = -\frac{1}{2}$

$r = \frac{38}{-76} = -\frac{1}{2}$

$a_n = a_1 \cdot r^{n-1}$

$a_n = 152\left(-\frac{1}{2}\right)^{n-1}$

$a_7 = 152\left(-\frac{1}{2}\right)^{7-1} = 152\left(-\frac{1}{2}\right)^6$

$= \frac{19}{8}$

EXAMPLE 3 **Write a rule given a term and common ratio**

One term of a geometric sequence is $a_4 = 12$. The common ratio is $r = 2$.

a. Write a rule for the nth term.

$a_n = a_1 \cdot r^{n-1}$

$12 = a_1 \cdot (2)^{4-1}$

$12 = a_1(2)^3$

$12 = a_1(8)$

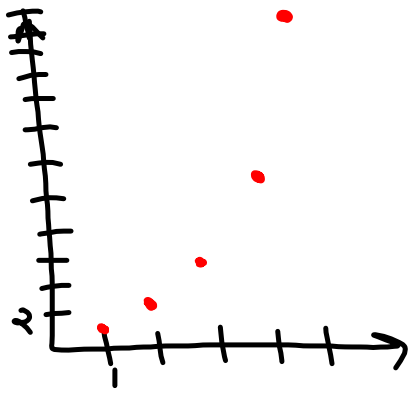
$\frac{12}{8} = a_1$

$a_1 = \frac{3}{2}$

$a_n = \frac{3}{2}(2)^{n-1}$

b. Graph the sequence.

$\frac{3}{2}, 3, 6, 12, 24$



12.3 Analyze Geometric Sequences and Series

EXAMPLE 4 **Write a rule given two terms**

Two terms of a geometric sequence are $a_3 = -48$ and $a_6 = 3072$. Find a rule for the nth term.

$a_n = a_1 \cdot r^{n-1}$

$-48 = a_1(-4)^{3-1}$

$-48 = a_1(-4)^2$

$\frac{-48}{16} = \frac{16a_1}{16}$

$-3 = a_1$

6 - 3

$-48 \cdot r \cdot r \cdot r = 3072$

$\frac{-48r^3}{-48} = \frac{3072}{-48}$

$\sqrt[3]{r^3} = \sqrt[3]{-64}$

$r = -4$

$a_n = -3(-4)^{n-1}$

EXAMPLE 4 **Write a rule given two terms**

Write a rule for the nth term of the geometric sequence. Then find a_8.

$a_2 = -12$ $a_4 = -3$

$4-2$

$-12 r r = -3$

$-12 r^2 = -3$

$\sqrt{r^2} = \frac{-3}{-12} = \sqrt{\frac{1}{4}}$

$r = \frac{1}{2}$

$a_n = a_1 r^{n-1}$

$-12 = a_1 \left(\frac{1}{2}\right)^{2-1}$

$2(-12) = a_1 \left(\frac{1}{2}\right) 2$

$-24 = a_1$

$a_n = -24\left(\frac{1}{2}\right)^{n-1}$

$a_8 = 24\left(\frac{1}{2}\right)^{8-1}$

$= 24\left(\frac{1}{2}\right)^7 = 24\left(\frac{1}{128}\right) = \frac{24}{128} = \frac{3}{16}$

IDENTIFYING GEOMETRIC SEQUENCES Tell whether the sequence is geometric. *Explain* why or why not.

7. $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$

$$r = \frac{1}{\frac{1}{2}} = 1 \cdot 2 = 2$$

$$r = \frac{\frac{3}{2}}{1} = \frac{3}{2}$$

No; no common ratio.

WRITING RULES Write a rule for the nth term of the geometric sequence. Then find a_7.

15. 1, −4, 16, −64, . . .

$a_1 = 1$

$r = \frac{-4}{1} = -4$

$a_n = a_1(r)^{n-1}$

$a_n = 1(-4)^{n-1}$

$a_n = (-4)^{n-1}$

$a_7 = (-4)^{7-1}$

$= (-4)^6$

$= 4096$

WRITING RULES Write a rule for the nth term of the geometric sequence. Then graph the first six terms of the sequence.

33. $a_4 = -12, r = -\frac{1}{4}$

$a_n = a_1 \cdot r^{n-1}$

$-12 = a_1\left(-\frac{1}{4}\right)^{4-1}$

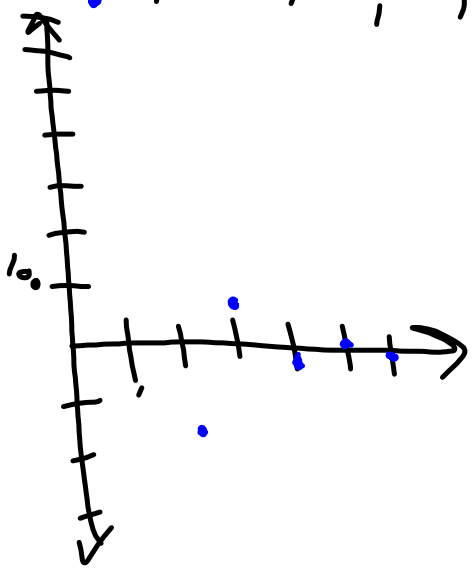
$-12 = a_1\left(-\frac{1}{4}\right)^{3}$

$-64 \cdot -12 = a_1\left(-\frac{1}{64}\right) \cdot -64$

$768 = a_1$

$a_n = 768\left(-\frac{1}{4}\right)^{n-1}$

$768, -192, 48, -12, 3, -\frac{3}{4}$



WRITING RULES Write a rule for the nth term of the geometric sequence that has the two given terms.

41. $a_1 = -\frac{1}{4}, a_4 = -16$

$a_n = a_1 r^{n-1}$

$-16 = \left(-\frac{1}{4}\right) r^{4-1}$

$-4(-16) = -\frac{1}{4} r^3 (-4)$

$\sqrt[3]{64} = \sqrt[3]{r^3}$

$r = 4$

$a_n = \left(-\frac{1}{4}\right)(4)^{n-1}$

WRITING RULES Write a rule for the nth term of the geometric sequence that has the two given terms.

45. $a_4 = 162, a_7 = 4374$

$a_n = a_1 r^{n-1}$

$\frac{162r^3}{162} = \frac{4374}{162}$

$\sqrt[3]{r^3} = \sqrt[3]{27}$

$r = 3$

$162 = a_1(3)^{4-1}$

$162 = a_1(3)^3$

$\frac{162}{27} = \frac{a_1(27)}{27}$

$6 = a_1$

$a_n = 6(3)^{n-1}$