12.4 Find Sums of Infinite Geometric Series

EXAMPLE 1 Find partial sums

Find and graph the partial sums S_n for $n = 1, 2, 3, 4,$ and 5.

Then describe what happens to S_n as n increases.

$r = \frac{1}{2}$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots$

$S_1 = \frac{1}{2} = 0.5$

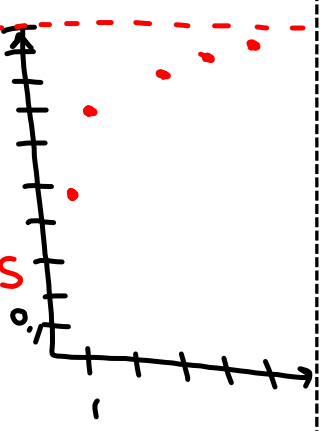
$S_2 = \frac{1}{2} + \frac{1}{4} = 0.75$

$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 0.875$

$S_4 = 0.9375$

$S_5 = 0.96875$

Sum = 1



$r = \frac{2}{5}$

$\frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \frac{16}{625} + \frac{32}{3125} + \cdots$

$S_1 = 0.4$

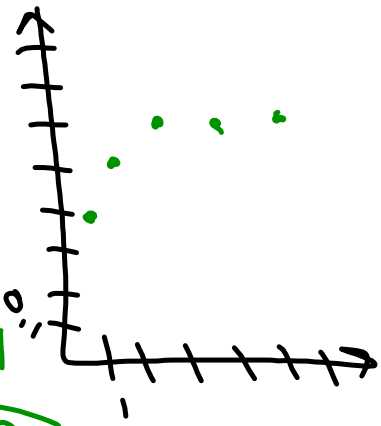
$S_2 = 0.56$

$S_3 = 0.624$

$S_4 = 0.6496$

$S_5 = 0.65984$

Sum = $\frac{2}{3}$



EXAMPLE 1 Find partial sums

Find and graph the partial sums S_n for $n = 1, 2, 3, 4,$ and 5.

Then describe what happens to S_n as n increases.

$r = \frac{1}{3}$

$2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \ldots$

$S_1 = 2$

$S_2 = 2.67$

$S_3 = 2.89$

$S_4 = 2.96$

$S_5 = 2.99$

Sum: 3

$r = -3$

$1 - 3 + 9 - 27 + \ldots ?$ 81

$S_1 = 1$

$S_2 = -2$

$S_3 = 7$

$S_4 = -20$

$S_5 = 61$

no sum

EXAMPLE 2 **Find sums of infinite geometric series**

Find the sum of the infinite geometric series, if it exists.

a. $\sum_{i=1}^{\infty} 5(0.8)^{i-1}$

$a_1 (r)^{n-1}$

$a_1 = 5$

$r = 0.8 = \frac{8}{10} = \frac{4}{5}$

$\frac{5}{1-\frac{4}{5}} = \frac{5}{\frac{1}{5}} = 5 \cdot 5$

$= 25$

b. $1 - \frac{3}{4} + \frac{9}{16} - \frac{27}{64} + \cdots$

$\frac{a_1}{1-r} = \frac{1}{1-(-\frac{3}{4})}$

$= \frac{1}{1+\frac{3}{4}} = \frac{1}{\frac{7}{4}} = \frac{4}{7}$

EXAMPLE 2 Find sums of infinite geometric series

Find the sum of the infinite geometric series, if it exists.

$$\sum_{n=1}^{\infty} 3\left(\frac{5}{4}\right)^{n-1}$$

$a_1(r)^{n-1}$

$a_1 = 3$

$r = \frac{5}{4}$

No sum

$$3 + \frac{3}{4} + \frac{3}{16} + \frac{3}{64} + \cdots$$

$a_1 = 3$

$r = \frac{1}{4}$

$$\frac{3}{1 - \frac{1}{4}} = \frac{3}{\frac{3}{4}} = 4$$

PARTIAL SUMS For the given series, find and graph the partial sums S_n for $n = 1, 2, 3, 4$, and 5. *Describe* what happens to S_n as n increases.

5. $4 + \frac{12}{5} + \frac{36}{25} + \frac{108}{125} + \frac{324}{625} + \cdots$

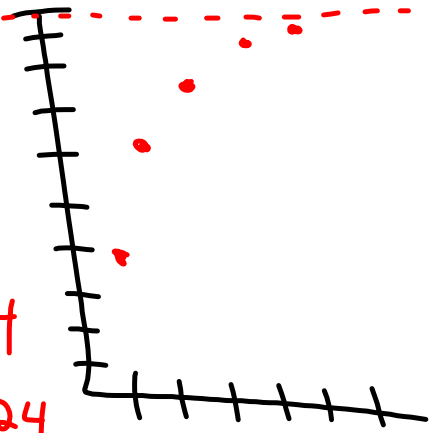
$S_1 = 4$

$S_2 = 6.4$

$S_3 = 7.84$

$S_4 = 8.704$

$S_5 = 9.2224$



6. $\frac{1}{4} + \frac{5}{4} + \frac{25}{4} + \frac{125}{4} + \frac{625}{4} + \cdots$

$r = 5$

$S_1 = \frac{1}{4} = 0.25$

$S_2 = 1.5$

$S_3 = 7.75$

$S_4 = 39$

$S_5 = 195.25$

FINDING SUMS Find the sum of the infinite geometric series, if it exists.

7. $\sum_{n=1}^{\infty} 8\left(\frac{1}{5}\right)^{n-1}$

$a_1 = 8$

$r = \frac{1}{5}$

$S_\infty = \frac{8}{1-\frac{1}{5}}$

$= \frac{8}{\frac{4}{5}} = 8\left(\frac{5}{4}\right)$

$= 10$

20. $-\frac{1}{8} - \frac{1}{12} - \frac{1}{18} - \frac{1}{27} + \cdots$

$a_1 = -\frac{1}{8}$ $r = -\frac{2}{3}$

$r = -\frac{2}{3}$

$S_\infty = \frac{-\frac{1}{8}}{1-\left(-\frac{2}{3}\right)}$

$= \frac{-\frac{1}{8}}{\frac{5}{3}} = -\frac{1}{8}\left(\frac{3}{5}\right)$

$= -\frac{3}{40}$