12.5 Use Recursive Rules with Sequences and Functions

$1, -2, 4, -8, \ldots$

$(-2)^0, (-2)^1, (-2)^2, (-2)^3$

Explicit Rule:

$$a_n = (-2)^{n-1}$$

Recursive Rule:

$a_1 = 1$

$a_2 = -2(1) = -2$

$a_3 = -2(-2) = 4$

$a_4 = -2(4) = -8$

$$a_n = -2(a_{n-1})$$
$$a_1 = 1$$

EXAMPLE 1 Evaluate recursive rules

Write the first six terms of the sequence.

a. $a_0 = 1, a_n = a_{n-1} + 4$

$a_0 = 1$

$a_1 = a_{1-1} + 4 = a_0 + 4 = 1 + 4 = 5$

$a_2 = a_{2-1} + 4 = a_1 + 4 = 5 + 4 = 9$

$a_3 = a_2 + 4 = 9 + 4 = 13$

$a_4 = 13 + 4 = 17$

$a_5 = 17 + 4 = 21$

1, 5, 9, 13, 17, 21

b. $a_1 = 1, a_n = 3a_{n-1}$

$a_1 = 1$

$a_2 = 3(a_1) = 3(1) = 3$

$a_3 = 3(3) = 9$

$a_4 = 3(9) = 27$

$a_5 = 3(27) = 81$

$a_6 = 3(81) = 243$

1, 3, 9, 27, 81, 243

EXAMPLE 1 **Evaluate recursive rules**

Write the first six terms of the sequence.

$a_1 = 4, a_n = 2a_{n-1} - 1$

$a_1 = 4$

$a_2 = 2(4) - 1 = 7$

$a_3 = 2(7) - 1 = 13$

$a_4 = = 25$

$a_5 = = 49$

$a_6 = = 97$

$a_0 = 1, a_n = a_{n-1} + n$

$a_0 = 1$

$a_1 = (1) + 1 = 2$

$a_2 = (2) + 2 = 4$

$a_3 = (4) + 3 = 7$

$a_4 = (7) + 4 = 11$

$a_5 = (11) + 5 = 16$

EXAMPLE 1 **Evaluate recursive rules**

Write the first six terms of the sequence.

$$a_1 = 6, \quad a_2 = 1, \quad a_n = a_{n-2} - a_{n-1}$$

$a_1 = 6$

$a_2 = 1$

$a_3 = a_1 - a_2 = 6 - 1 = 5$

$a_4 = a_2 - a_3 = 1 - 5 = -4$

$a_5 = a_3 - a_4 = 5 - (-4) = 9$

$a_6 = a_4 - a_5 = -4 - 9 = -13$

EXAMPLE 2 Write recursive rules

Write a recursive rule for the sequence.

a. 3, 13, 23, 33, 43, . . .

+10 +10 +10 +10

$a_1 = 3$

$a_2 = 3 + 10 = a_1 + 10$

$a_3 = 13 + 10 = a_2 + 10$

$a_n = a_{n-1} + 10$

$a_1 = 3$

b. 16, 40, 100, 250, 625, . . .

$\cdot\frac{5}{2}$ $\cdot\frac{5}{2}$ $\cdot\frac{5}{2}$ $\cdot\frac{5}{2}$

$a_n = \frac{5}{2}(a_{n-1})$

$a_1 = 16$

EXAMPLE 3 **Write recursive rules for special sequences**

Write a recursive rule for the sequence.

a. 1, 1, 2, 3, 5, . . . 8, 13, 21,

+1 +1 +2 +3 5

$a_n = a_{n-1} + a_{n-2}$

$a_1 = 1$

$a_2 = 1$

b. 1, 1, 2, 6, 24, . . .

$a_0 = 1$

$a_1 = 1(1) = 1$

$a_2 = 2(1) = 2$

$a_3 = 3(2) = 6$

$a_4 = 4(6) = 24$

$a_n = n(a_{n-1})$

$a_0 = 1$

WRITING TERMS Write the first five terms of the sequence.

9. $a_1 = 2$
$a_n = n^2 + 3n - a_{n-1}$

$a_1 = 2$

$a_2 = 2^2 + 3(2) - (2) = 8$

$a_3 = 3^2 + 3(3) - (8) = 10$

$a_4 = 4^2 + 3(4) - (10) = 18$

$a_5 = 5^2 + 3(5) - 18 = 22$

WRITING RULES Write a recursive rule for the sequence. The sequence may be arithmetic, geometric, or neither.

20. 3, 5, 15, 75, 1125, . . .

$a_1 = 3$

$a_2 = 5$

$a_3 = 3 \cdot 5 = 15 = a_1 \cdot a_2$

$a_4 = 5 \cdot 15 = 75 = a_2 \cdot a_3$

$a_n = (a_{n-2})(a_{n-1})$

$a_1 = 3$

$a_2 = 5$

neither