

## 12.5 Use Recursive Rules with Sequences and Functions

Explicit Rule:

$$a_n = (-2)^{n-1}$$

<sup>1</sup> 1, <sup>2</sup>-2, <sup>3</sup>4, <sup>4</sup>-8, ...  
 $(-2)^0, (-2)^1, (-2)^2, (-2)^3$

Recursive Rule:

$$\begin{aligned} a_1 &= 1 \\ a_2 &= -2(1) = -2 \\ a_3 &= -2(-2) = 4 \\ a_4 &= -2(4) = -8 \end{aligned}$$

$$\begin{aligned} a_n &= -2(a_{n-1}) \\ a_1 &= 1 \end{aligned}$$

**EXAMPLE 1** Evaluate recursive rules

Write the first six terms of the sequence.

a.  $a_0 = 1, a_n = a_{n-1} + 4$

$a_0 = 1$

$a_1 = a_{1-1} + 4 = a_0 + 4 = 1 + 4 = 5$

$a_2 = a_{2-1} + 4 = a_1 + 4 = 5 + 4 = 9$

$a_3 = a_2 + 4 = 9 + 4 = 13$

$a_4 = 13 + 4 = 17$

$a_5 = 17 + 4 = 21$

1, 5, 9, 13, 17, 21

b.  $a_1 = 1, a_n = 3a_{n-1}$

$a_1 = 1$

$a_2 = 3(a_1) = 3(1) = 3$

$a_3 = 3(3) = 9$

$a_4 = 3(9) = 27$

$a_5 = 3(27) = 81$

$a_6 = 3(81) = 243$

1, 3, 9, 27, 81, 243

**EXAMPLE 1** Evaluate recursive rules

Write the first six terms of the sequence.

$$a_1 = 4, a_n = 2a_{n-1} - 1$$

$$\begin{aligned}
 a_1 &= 4 \\
 a_2 &= 2(4) - 1 = 7 \\
 a_3 &= 2(7) - 1 = 13 \\
 a_4 &= 25 \\
 a_5 &= 49 \\
 a_6 &= 97
 \end{aligned}$$

$$a_0 = 1, a_n = a_{n-1} + n$$

$$\begin{aligned}
 a_0 &= 1 \\
 a_1 &= (1) + 1 = 2 \\
 a_2 &= (2) + 2 = 4 \\
 a_3 &= (4) + 3 = 7 \\
 a_4 &= (7) + 4 = 11 \\
 a_5 &= (11) + 5 = 16
 \end{aligned}$$

**EXAMPLE 1** Evaluate recursive rules

Write the first six terms of the sequence.

$$a_1 = 6, \quad a_2 = 1, \quad a_n = a_{n-2} - a_{n-1}$$

$$a_1 = 6$$

$$a_2 = 1$$

$$a_3 = a_1 - a_2 = 6 - 1 = 5$$

$$a_4 = a_2 - a_3 = 1 - 5 = -4$$

$$a_5 = a_3 - a_4 = 5 - (-4) = 9$$

$$a_6 = a_4 - a_5 = -4 - 9 = -13$$

**EXAMPLE 2** Write recursive rules

Write a recursive rule for the sequence.

a. 3, 13, 23, 33, 43, ...

$$\begin{array}{cccc} \vee & \vee & \vee & \vee \\ +10 & +10 & +10 & +10 \end{array}$$

$$a_1 = 3$$

$$a_2 = 3 + 10 = a_1 + 10$$

$$a_3 = 13 + 10 = a_2 + 10$$

$$\begin{array}{l} a_n = a_{n-1} + 10 \\ a_1 = 3 \end{array}$$

b. 16, 40, 100, 250, 625, ...

$$\begin{array}{cccc} \cdot \frac{5}{2} & \cdot \frac{5}{2} & \cdot \frac{5}{2} & \cdot \frac{5}{2} \end{array}$$

$$\begin{array}{l} a_n = \frac{5}{2} a_{n-1} \\ a_1 = 16 \end{array}$$

**EXAMPLE 3** Write recursive rules for special sequences

Write a recursive rule for the sequence.

a. 1, 1, 2, 3, 5, **8, 13, 21,**  
 $\checkmark \checkmark \checkmark \checkmark \checkmark$   
 $+1 +1 +2 +3 +5$

$$a_n = a_{n-1} + a_{n-2}$$

$$a_1 = 1$$

$$a_2 = 1$$

b. 1, 1, 2, 6, 24, ...

$$a_0 = 1$$

$$a_1 = 1(1) = 1$$

$$a_2 = 2(1) = 2$$

$$a_3 = 3(2) = 6$$

$$a_4 = 4(6) = 24$$

$$a_n = n(a_{n-1})$$

$$a_0 = 1$$

**WRITING TERMS** Write the first five terms of the sequence.

9.  $a_1 = 2$

$$a_n = n^2 + 3n - a_{n-1}$$

$a_1 = 2$

$a_2 = 2^2 + 3(2) - (2) = 8$

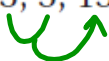
$a_3 = 3^2 + 3(3) - (8) = 10$

$a_4 = 4^2 + 3(4) - (10) = 18$

$a_5 = 5^2 + 3(5) - 18 = 22$

**WRITING RULES** Write a recursive rule for the sequence. The sequence may be arithmetic, geometric, or neither.

20. 3, 5, 15, 75, 1125, ...



$$a_1 = 3$$

$$a_2 = 5$$

$$a_3 = 3 \cdot 5 = 15 = a_1 \cdot a_2$$

$$a_4 = 5 \cdot 15 = 75 = a_2 \cdot a_3$$

$$a_n = (a_{n-2})(a_{n-1})$$

$$a_1 = 3$$

$$a_2 = 5$$

neither