

3.6 Multiply Matrices

Matrix Review

$$A = \begin{bmatrix} 3 & -7 \\ 1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} -10 & 2 \\ 4 & -8 \end{bmatrix}, \quad C = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

2×2
 2×2
 2×1

Find the following:

AB

$$\begin{bmatrix} \frac{-58}{r_1c_1} & \frac{62}{r_1c_2} \\ \frac{10}{r_2c_1} & \frac{-38}{r_2c_2} \end{bmatrix}$$

CB

2×1 2×2

$$\boxed{\text{DNE}}$$

BC

$$\begin{bmatrix} \frac{-34}{r_1c_1} \\ \frac{28}{r_2c_1} \end{bmatrix}$$

EXAMPLE 3 Use matrix operations

Using the given matrices, evaluate the expression.

$$A = \begin{bmatrix} 4 & 3 \\ -1 & -2 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} -3 & 0 \\ 1 & -2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 \\ -3 & -1 \end{bmatrix}$$

a. $A(B + C)$

$$\begin{bmatrix} 4 & 3 \\ -1 & -2 \\ 2 & 0 \end{bmatrix} \left(\begin{bmatrix} -3 & 0 \\ 1 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ -3 & -1 \end{bmatrix} \right)$$

$$\begin{bmatrix} 4 & 3 \\ -1 & -2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} -2 & 4 \\ -2 & -3 \end{bmatrix}$$

$$\begin{bmatrix} -14 & 7 \\ 6 & 2 \\ 14 & 18 \end{bmatrix}$$

b. $AB + AC$

$$\begin{bmatrix} -9 & -6 \\ 1 & 4 \\ -6 & 10 \end{bmatrix} + \begin{bmatrix} -5 \\ 15 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} -14 & 7 \\ 6 & 2 \\ 14 & 18 \end{bmatrix}$$

EXAMPLE 3 Use matrix operations

Using the given matrices, evaluate the expression.

$$A = \begin{bmatrix} -1 & 2 \\ -3 & 0 \\ 4 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix}, C = \begin{bmatrix} -4 & 5 \\ 1 & 0 \end{bmatrix}$$

$$AB - AC = A(B - C)$$

$$\begin{bmatrix} -1 & 2 \\ -3 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ -3 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} -4 & 5 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -7 & -4 \\ -9 & -6 \\ 10 & 1 \end{bmatrix} - \begin{bmatrix} 6 & -5 \\ 12 & -5 \\ 15 & 20 \end{bmatrix}$$

$$\begin{bmatrix} -13 & 1 \\ -21 & 9 \\ 25 & -13 \end{bmatrix}$$

EXAMPLE 3 Use matrix operations

Using the given matrices, evaluate the expression.

$$A = \begin{bmatrix} -1 & 2 \\ -3 & 0 \\ 4 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix}, C = \begin{bmatrix} -4 & 5 \\ 1 & 0 \end{bmatrix}$$

$$-\frac{1}{2}(AB)$$

$$\begin{bmatrix} \frac{1}{2} & 2 \\ \frac{1}{2} & 3 \\ -5 & -\frac{7}{2} \end{bmatrix}$$

EXAMPLE 4 Use matrices to calculate total cost

SPORTS Two hockey teams submit equipment lists for the season as shown.

Each stick costs \$60, each puck costs \$2, and each uniform costs \$35. Use matrix multiplication to find the total cost of equipment for each team.

EQUIPMENT LISTS

Women's Team

14 sticks
30 pucks
18 uniforms

Men's Team

16 sticks
25 pucks
20 uniforms

$$\begin{matrix} W. \\ M. \end{matrix} \begin{matrix} S & P & U \\ \left[\begin{array}{ccc} 14 & 30 & 18 \\ 16 & 25 & 20 \end{array} \right] \end{matrix} \cdot \begin{matrix} S \\ P \\ U \\ \left[\begin{array}{c} 60 \\ 2 \\ 35 \end{array} \right] \end{matrix}$$

2×3 3×1

$$\begin{bmatrix} 14(60) + 30(2) + 18(35) \\ 16(60) + 25(2) + 20(35) \end{bmatrix} = \begin{bmatrix} 1530 \\ 1710 \end{bmatrix} \begin{matrix} W \\ M \end{matrix}$$

$W = \$1530$

$M = \$1710$

EVALUATING EXPRESSIONS Using the given matrices, evaluate the expression.

$$A = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix}, C = \begin{bmatrix} -6 & 3 \\ 4 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 3 & 2 \\ -3 & 1 & 4 \\ 2 & 1 & -2 \end{bmatrix}, E = \begin{bmatrix} -3 & 1 & 4 \\ 7 & 0 & -2 \\ 3 & 4 & -1 \end{bmatrix}$$

22. $3AB$

$$3 \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix} = 3 \begin{bmatrix} -12 & 11 \\ 16 & -10 \end{bmatrix}$$
$$\begin{bmatrix} -36 & 33 \\ 48 & -30 \end{bmatrix}$$

EVALUATING EXPRESSIONS Using the given matrices, evaluate the expression.

$$A = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix}, C = \begin{bmatrix} -6 & 3 \\ 4 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 3 & 2 \\ -3 & 1 & 4 \\ 2 & 1 & -2 \end{bmatrix}, E = \begin{bmatrix} -3 & 1 & 4 \\ 7 & 0 & -2 \\ 3 & 4 & -1 \end{bmatrix}$$

27. $(D + E)D$

$$\begin{pmatrix} -2 & 4 & 6 \\ 4 & 1 & 2 \\ 5 & 5 & -3 \end{pmatrix} \begin{pmatrix} 1 & 3 & 2 \\ -3 & 1 & 4 \\ 2 & 1 & -2 \end{pmatrix} = \begin{pmatrix} -2 & 4 & 0 \\ -4 & 15 & 8 \\ -16 & 17 & 36 \end{pmatrix}$$