

5.4 Factor and Solve Polynomial Equations

EXAMPLE 1 Find a common monomial factor

Factor the polynomial completely.

a. $x^3 + 2x^2 - 15x$

$$x(x^2 + 2x - 15)$$
$$x(x+5)(x-3)$$

(Handwritten red work: An arrow points from the constant term -15 in the quadratic to the factors 5 and -3 in the binomials.)

b. $2y^5 - 18y^3$

$$2y^3(y^2 - 9)$$
$$2y^3(y+3)(y-3)$$

(Handwritten green work: The final factored form is enclosed in a green box.)

Factoring Sums or Differences of Cubes (SOAP)

$$a^3 + b^3 = \boxed{(a+b)(a^2 - ab + b^2)}$$

The equation is written in red ink. Above the first factor $(a+b)$ is a red 'S'. Above the second factor $(a^2 - ab + b^2)$ are red 'O' and 'AP'.

$$a^3 - b^3 = \boxed{(a-b)(a^2 + ab + b^2)}$$

The equation is written in blue ink. Above the first factor $(a-b)$ is a blue 'S'. Above the second factor $(a^2 + ab + b^2)$ are blue 'O' and 'AP'.

EXAMPLE 2 Factor the sum or difference of two cubes

Factor the polynomial completely.

a. $x^3 + 64$

$$\left(\underbrace{x}_{\text{S}}\right)^3 + \left(\underbrace{4}_{\text{AP}}\right)^3$$

$$\boxed{(x+4)(x^2-4x+16)}$$

b. $16z^5 - 250z^2$

$$2z^2(8z^3 - 125)$$

$$2z^2\left(\underbrace{(2z)}_{\text{S}}\right)^3 - \left(\underbrace{5}_{\text{AP}}\right)^3$$

$$\boxed{2z^2(2z-5)(4z^2+10z+25)}$$

EXAMPLE 3 Factor by grouping

Factor the polynomial $x^3 - 3x^2 - 16x + 48$ completely.

$$\begin{aligned} & x^3 - 3x^2 - 16x + 48 \\ & x^2(x-3) - 16(x-3) \\ & (x-3)(x^2-16) \\ & (x-3)(x-4)(x+4) \end{aligned}$$

EXAMPLE 4 Factor polynomials in quadratic form

Factor completely

a. $16x^4 - 81$

$$(4x^2)^2 - (9)^2$$

$$(4x^2 - 9)(4x^2 + 9)$$

$$((2x)^2 - (3)^2)(4x^2 + 9)$$

$$(2x - 3)(2x + 3)(4x^2 + 9)$$

b. $2p^8 + 10p^5 + 12p^2$

$$2p^2(p^6 + 5p^3 + 6)$$

$$u = p^3$$

$$2p^2(u^2 + 5u + 6)$$

$$2p^2(u + 2)(u + 3)$$

$$2p^2(p^3 + 2)(p^3 + 3)$$

EXAMPLE 5

What are the real-number solutions of the equation $3x^5 + 15x = 18x^3$?

$$3x^5 + 15x = 18x^3$$

$$\begin{array}{r} -18x^3 \\ -18x^3 \end{array}$$

$$3x^5 - 18x^3 + 15x = 0$$

$$3x(x^4 - 6x^2 + 5) = 0$$

$$3x(u^2 - 6u + 5) = 0$$

$u = x^2$

$$3x(u-1)(u-5) = 0$$

$$3x(x^2-1)(x^2-5) = 0$$

$$3x(x-1)(x+1)(x^2-5) = 0$$

$$\begin{array}{cccc} 3x=0 & x-1=0 & x+1=0 & x^2-5=0 \\ \boxed{x=0} & \boxed{x=1} & \boxed{x=-1} & \sqrt{x^2} = \sqrt{5} \\ & & & \boxed{x = \pm\sqrt{5}} \end{array}$$

EXAMPLE 5

Find the real-number solutions of the equation.

$$4x^5 - 40x^3 + 36x = 0$$

EXAMPLE 5

Find the real-number solutions of the equation.

$$-27x^3 + 15x^2 = -6x^4$$