

5.5 Apply the Remainder and Factor Theorems

EXAMPLE 1 Use polynomial long division

Divide $f(x) = 3x^4 - 5x^3 + 4x - 6$ by $x^2 - 3x + 5$.

$$\begin{array}{r}
 \boxed{x^2 - 3x + 5} \overline{) 3x^4 - 5x^3 + 0x^2 + 4x - 6} \\
 \underline{-3x^4 + 9x^3 + 15x^2} \\
 \hline
 \quad \quad \quad 4x^3 - 15x^2 + 4x \\
 \underline{-4x^3 + 12x^2 + 20x} \\
 \hline
 \quad \quad \quad -3x^2 - 16x - 6 \\
 \underline{+3x^2 + 9x + 15} \\
 \hline
 \quad \quad \quad \boxed{-25x + 9}
 \end{array}$$

The quotient is $3x^2 + 4x - 3$ and the remainder is $-25x + 9$.

EXAMPLE 1 Use polynomial long division

Divide using polynomial long division.

$$(2x^4 + x^3 + x - 1) \div (x^2 + 2x - 1)$$

Handwritten polynomial long division diagram:

$$\begin{array}{r} 2x^2 - 3x + 8 \\ \hline x^2 + 2x - 1 \overline{)2x^4 + x^3 + 0x^2 + x - 1} \\ -2x^4 - 4x^3 \\ \hline -3x^3 + 2x^2 + x \\ +3x^3 + 6x^2 + 3x \\ \hline 8x^2 - 2x - 1 \\ -8x^2 + 16x + 8 \\ \hline -18x + 7 \end{array}$$

The quotient is $2x^2 - 3x + 8$ and the remainder is $-18x + 7$.

EXAMPLE 2 Use polynomial long division with a linear divisor

Divide $f(x) = x^3 + 5x^2 - 7x + 2$ by $x - 2$.

$$\begin{array}{r}
 x-2 \overline{)x^3 + 5x^2 - 7x + 2} \\
 \underline{-x^3 + 2x^2} \\
 \hline
 7x^2 - 7x \\
 \underline{-7x^2 + 14x} \\
 \hline
 7x + 2 \\
 \underline{-7x + 14} \\
 \hline
 16
 \end{array}$$

The quotient is $x^2 + 7x + 7$ and the remainder is 16.

EXAMPLE 3 Use synthetic divisionDivide $f(x) = x^3 + 5x^2 - 7x + 2$ by $x - 2$.

$$\begin{aligned}x-2 &= 0 \\x &= 2\end{aligned}$$

2 | 1 5 -7 2

↓ 2 14 14

$x^2 + 7x + 7 + \frac{16}{x-2}$

EXAMPLE 3 Use synthetic division

Divide $f(x) = 2x^3 + x^2 - 8x + 5$ by $x + 3$ using synthetic division.

The diagram shows the synthetic division process. The divisor is $x + 3$, with $x = -3$ written above it. The dividend is $2x^3 + x^2 - 8x + 5$. The quotient is $2x^2 - 5x + 7$ and the remainder is -16 .

-3	2	1	-8	5
	\downarrow	\vdots	\vdots	\vdots
	-6	15	-21	
	\nearrow	\nearrow	\nearrow	
	2	-5	7	-16
	$2x^2 - 5x + 7 + \frac{-16}{x+3}$			

USING LONG DIVISION Divide using polynomial long division.

7. $(3x^3 + 11x^2 + 4x + 1) \div (x^2 + x)$

The diagram illustrates the polynomial long division of $3x^3 + 11x^2 + 4x + 1$ by $x^2 + x$. The quotient is $3x + 8$, and the remainder is $-4x + 1$.

Step-by-step breakdown:

- Setup:** The divisor $x^2 + x$ is written below the dividend $3x^3 + 11x^2 + 4x + 1$.
- First Division:** The first term of the dividend, $3x^3$, is divided by the first term of the divisor, x^2 , resulting in $3x$. This is written above the division bar.
- Multiplication:** $3x$ is multiplied by the divisor $x^2 + x$ to get $3x^3 + 3x^2$. This is written under the first two terms of the dividend.
- Subtraction:** $3x^3 + 3x^2$ is subtracted from $3x^3 + 11x^2$ to get $8x^2$. A dashed line connects this result to the next step.
- Second Division:** The new dividend term, $8x^2$, is divided by x^2 , resulting in 8 . This is written above the division bar.
- Multiplication:** 8 is multiplied by the divisor $x^2 + x$ to get $8x^2 + 8x$. This is written under the next two terms of the dividend.
- Subtraction:** $8x^2 + 8x$ is subtracted from $4x + 1$ to get $-4x + 1$. A dashed line connects this result to the final answer.
- Final Answer:** The quotient is $3x + 8$, and the remainder is $-4x + 1$.

5.5 Apply the Remainder and Factor Theorems

EXAMPLE 4 Factor a polynomial

Factor the polynomial completely given that $x - 4$ is a factor.

$$f(x) = x^3 - 6x^2 + 5x + 12$$

$$x - 4 = 0$$

$$x = 4$$

Diagram illustrating the synthetic division process:

- The divisor is $x - 4$.
- The dividend coefficients are 1, -6, 5, 12.
- The quotient is $x^2 - 2x - 3$.
- The remainder is 0.
- The factors are $(x - 4)$ and $(x^2 - 2x - 3)$.
- The final factored form is $(x - 4)(x - 3)(x + 1)$.

EXAMPLE 4 Factor a polynomial

Factor $f(x) = 3x^3 - 4x^2 - 28x - 16$ completely given that $x + 2$ is a factor.

$$\begin{array}{r}
 \begin{array}{ccccccccc}
 & 3 & -4 & -28 & -16 & & & \\
 -2 & | & & & & & & x=-2 \\
 & \downarrow & & & & & & \\
 & -6 & & 20 & 16 & & & \\
 & \nearrow & \nearrow & \nearrow & \nearrow & & & \\
 3 & -10 & -8 & 0 & & & & \\
 \end{array} \\
 (x+2)(3x^2 - 10x - 8) \\
 AC = -24 \\
 3x^2 - 12x - 2 \\
 \quad \quad \quad \quad 2x + 2x - 8 \\
 3x(x-4) + 2(x-4) \\
 (x+2)(x-4)(3x+2)
 \end{array}$$

EXAMPLE 5

One zero of $f(x) = x^3 - 2x^2 - 23x + 60$ is $x = 3$. What is another zero of f ?

$$\begin{array}{r} 3 \quad | \quad 1 \quad -2 \quad -23 \quad 60 \\ \downarrow \quad | \quad 3 \quad 3 \quad -60 \\ 1 \quad | \quad 1 \quad -20 \quad 0 \end{array}$$

$$\begin{aligned} x^2 + x - 20 &= 0 \\ (x+5)(x-4) &= 0 \\ x = -5 &\qquad x = 4 \end{aligned}$$

$$x = 3, 4, -5$$

EXAMPLE 5

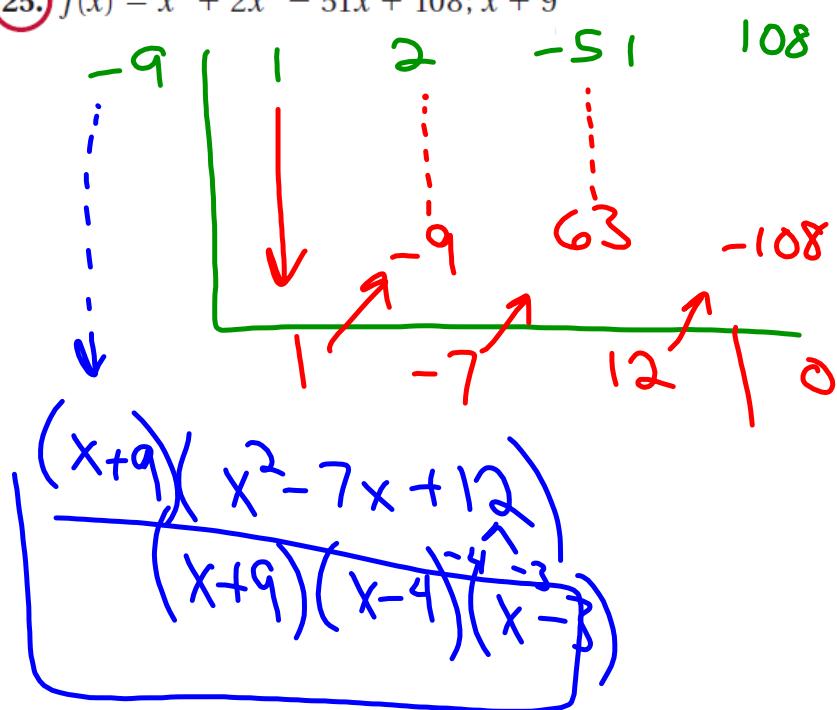
Find the other zeros of f given that $f(-2) = 0$.

$$f(x) = x^3 + 2x^2 - 9x - 18$$

$$x = -2$$

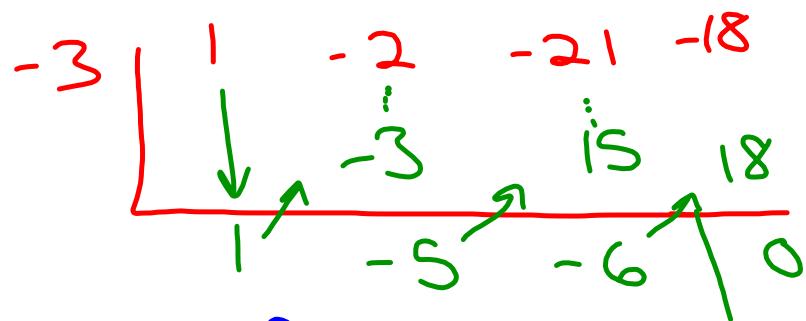
FACTOR Given polynomial $f(x)$ and a factor of $f(x)$, factor $f(x)$ completely.

25. $f(x) = x^3 + 2x^2 - 51x + 108; x + 9$



FIND ZEROS Given polynomial function f and a zero of f , find the other zeros.

29. $f(x) = x^3 - 2x^2 - 21x - 18; -3$



$$x^2 - 5x - 6 = 0$$

$$(x-6)(\overset{-6}{x+1}) = 0$$

$$x=6 \quad x=-1$$

$$x = -3, -1, 6$$