

5.5 Apply the Remainder and Factor Theorems

EXAMPLE 1 Use polynomial long division

Divide $f(x) = 3x^4 - 5x^3 + 4x - 6$ by $x^2 - 3x + 5$.

$$\begin{array}{r}
 x^2 - 3x + 5 \overline{) 3x^4 - 5x^3 + 0x^2 + 4x - 6} \\
 \underline{-3x^4 + 9x^3 - 15x^2} \\
 4x^3 - 15x^2 + 4x \\
 \underline{-4x^3 + 12x^2 + 20x} \\
 -3x^2 - 16x - 6 \\
 \underline{+3x^2 + 9x + 15} \\
 -25x + 9
 \end{array}$$

$3x^2 + 4x - 3 + \frac{-25x + 9}{x^2 - 3x + 5}$

EXAMPLE 1 Use polynomial long division

Divide using polynomial long division.

$$(2x^4 + x^3 + x - 1) \div (x^2 + 2x - 1)$$

$$\begin{array}{r}
 x^2 + 2x - 1 \overline{) 2x^4 + x^3 + 0x^2 + x - 1} \\
 \underline{-2x^4 + 4x^3 + 2x^2} \\
 -3x^3 + 2x^2 + x \\
 \underline{+3x^3 + 6x^2 + 3x} \\
 8x^2 - 2x - 1 \\
 \underline{-8x^2 + 16x + 8} \\
 -18x + 7
 \end{array}$$

$2x^2 - 3x + 8 + \frac{-18x + 7}{x^2 + 2x - 1}$

EXAMPLE 2 Use polynomial long division with a linear divisorDivide $f(x) = x^3 + 5x^2 - 7x + 2$ by $x - 2$.

The image shows a handwritten polynomial long division of $x^3 + 5x^2 - 7x + 2$ by $x - 2$. The divisor $x - 2$ is written on the left. The dividend $x^3 + 5x^2 - 7x + 2$ is written in blue. The first step shows subtracting $-x^3 + 2x^2$ (written in red) from the dividend, leaving a remainder of $7x^2 - 7x + 2$. The next step shows subtracting $-7x^2 + 14x$ (written in red) from the current remainder, leaving $7x + 2$. The final step shows subtracting $-7x + 14$ (written in red) from $7x + 2$, leaving a remainder of 16 . The final quotient is $x^2 + 7x + 7 + \frac{16}{x-2}$, which is boxed in red. A green arrow points from the circled remainder 16 to the fraction part of the quotient.

$$\begin{array}{r} x-2 \overline{) x^3 + 5x^2 - 7x + 2} \\ \underline{-x^3 + 2x^2} \\ 7x^2 - 7x + 2 \\ \underline{-7x^2 + 14x} \\ 7x + 2 \\ \underline{-7x + 14} \\ 16 \end{array}$$
$$x^2 + 7x + 7 + \frac{16}{x-2}$$

EXAMPLE 3 Use synthetic divisionDivide $f(x) = x^3 + 5x^2 - 7x + 2$ by $x - 2$.

$$\begin{aligned}x - 2 &= 0 \\x &= 2\end{aligned}$$

$$\begin{array}{r|rrrr} 2 & 1 & 5 & -7 & 2 \\ & \downarrow & \vdots & \vdots & \\ & & 2 & 14 & 14 \\ \hline & 1 & 7 & 7 & 16 \\ & \downarrow & \downarrow & \downarrow & \downarrow \\ & x^2 & + 7x & + 7 & + \frac{16}{x-2} \end{array}$$

EXAMPLE 3 Use synthetic divisionDivide $f(x) = 2x^3 + x^2 - 8x + 5$ by $x + 3$ using synthetic division.

$x+3=0$
 $\therefore x=-3$

-3	2	1	-8	5
	\downarrow	\vdots	\vdots	\vdots
	-6	15	-21	
	\hline	\hline	\hline	\hline
	2	-5	7	-16
	\vdots	\vdots	\vdots	

$$2x^2 - 5x + 7 + \frac{-16}{x+3}$$

USING LONG DIVISION Divide using polynomial long division.

7. $(3x^3 + 11x^2 + 4x + 1) \div (x^2 + x)$

The image shows a handwritten polynomial long division. On the left, the divisor $x^2 + x + 0$ is written in green. To its right, the dividend $3x^3 + 11x^2 + 4x + 1$ is written in green. A blue line is drawn under the dividend. Below this line, the first subtraction step is shown in red: $-3x^3 + 3x^2$. A blue line is drawn under this subtraction. Below that, the second subtraction step is shown in blue: $8x^2 + 4x$. Below this, the third subtraction step is shown in red: $-8x^2 + 8x$. A green line is drawn under this subtraction. Below that, the remainder $-4x + 1$ is written in green and circled in red. To the right of the main division, a red box contains the final answer: $3x + 8 + \frac{-4x + 1}{x^2 + x}$. Blue dashed arrows point from the boxed answer back to the corresponding terms in the long division process.

$$\begin{array}{r}
 \underline{x^2 + x + 0} \overline{) 3x^3 + 11x^2 + 4x + 1} \\
 \underline{-3x^3 + 3x^2} \\
 8x^2 + 4x \\
 \underline{-8x^2 + 8x} \\
 -4x + 1
 \end{array}$$

$3x + 8 + \frac{-4x + 1}{x^2 + x}$

5.5 Apply the Remainder and Factor Theorems

EXAMPLE 4 Factor a polynomial

Factor the polynomial completely given that $x - 4$ is a factor.

$$f(x) = x^3 - 6x^2 + 5x + 12$$

$$\begin{aligned} x - 4 &= 0 \\ x &= 4 \end{aligned}$$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 5 & 12 \\ & & 4 & -8 & -12 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

$$\frac{(x-4)(x^2 - 2x - 3)}{(x-4)(x-3)(x+1)}$$

EXAMPLE 4 Factor a polynomialFactor $f(x) = 3x^3 - 4x^2 - 28x - 16$ completely given that $x + 2$ is a factor.

$x = -2$

-2	3	-4	-28	-16	
	↓	⋮	⋮		
	3	-6	20	16	
	↗	↗	↗	↗	
	3	-10	-8	0	

$(x+2)(3x^2 - 10x - 8)$

$AC = -24$

$3x^2 - 12x + 2x - 8$

$3x(x-4) + 2(x-4)$

$(x+2)(x-4)(3x+2)$

EXAMPLE 5

One zero of $f(x) = x^3 - 2x^2 - 23x + 60$ is $x = 3$. What is another zero of f ?

$$\begin{array}{r|rrrr} 3 & 1 & -2 & -23 & 60 \\ & \downarrow & \uparrow & \uparrow & \uparrow \\ & & 3 & 3 & -60 \\ & & & -20 & 0 \end{array}$$

$$\begin{aligned} x^2 + x - 20 &= 0 \\ (x+5)(x-4) &= 0 \\ x &= -5 \quad x = 4 \end{aligned}$$

$$x = 3, 4, -5$$

EXAMPLE 5

Find the other zeros of f given that $f(-2) = 0$.

$$f(x) = x^3 + 2x^2 - 9x - 18$$

$$x = -2$$

FACTOR Given polynomial $f(x)$ and a factor of $f(x)$, factor $f(x)$ completely.

25. $f(x) = x^3 + 2x^2 - 51x + 108; x + 9$

$(x+9)(x^2 - 7x + 12)$
 $(x+9)(x-4)(x-3)$

