5.5 Apply the Remainder and Factor Theorems

EXAMPLE 1 Use polynomial long division

Divide $f(x) = 3x^4 - 5x^3 + 4x - 6$ by $x^2 - 3x + 5$.

$$\boxed{3x^2 + 4x - 3 + \frac{-25x + 9}{x^2 - 3x + 5}}$$

$$x^2 - 3x + 5 \,\big)\, 3x^4 - 5x^3 + 0x^2 + 4x - 6$$

$$\underline{-\cancel{3x^4} + 9x^3 - 15x^2}$$

$$\cancel{4x^3} - 15x^2 + 4x$$

$$\underline{-\cancel{4x^3} + 12x^2 - 20x}$$

$$-\cancel{3x^2} - 16x - 6$$

$$\underline{+\cancel{3x^2} - 9x - 15}$$

$$-25x + 9$$

EXAMPLE 1 **Use polynomial long division**

Divide using polynomial long division.

$(2x^4 + x^3 + x - 1) \div (x^2 + 2x - 1)$

$$2x^2 - 3x + 8 + \frac{-18x+7}{x^2+2x-1}$$

$$x^2+2x-1 \overline{)\, 2x^4 + x^3 + 0x^2 + x - 1}$$

$$-\,(2x^4 + 4x^3 - 2x^2)$$

$$-3x^3 + 2x^2 + x$$

$$-\,(-3x^3 - 6x^2 + 3x)$$

$$8x^2 - 2x - 1$$

$$-\,(8x^2 + 16x - 8)$$

$$-18x + 7$$

EXAMPLE 2 Use polynomial long division with a linear divisor

Divide $f(x) = x^3 + 5x^2 - 7x + 2$ by $x - 2$.

$$\boxed{x^2 + 7x + 7 + \frac{16}{x-2}}$$

$$
\begin{array}{r|l}
 & x^2 + 7x + 7 \\
\hline
x - 2 & x^3 + 5x^2 - 7x + 2 \\
 & -(x^3 - 2x^2) \\
\hline
 & 7x^2 - 7x \\
 & -(7x^2 - 14x) \\
\hline
 & 7x + 2 \\
 & -(7x - 14) \\
\hline
 & 16
\end{array}
$$

EXAMPLE 3 **Use synthetic division**

Divide $f(x) = x^3 + 5x^2 - 7x + 2$ by $x - 2$.

$x - 2 = 0$
$x = 2$

$$\begin{array}{r|rrrr} 2 & 1 & 5 & -7 & 2 \\ & & 2 & 14 & 14 \\ \hline & 1 & 7 & 7 & 16 \end{array}$$

$$\boxed{x^2 + 7x + 7 + \frac{16}{x-2}}$$

EXAMPLE 3 Use synthetic division

Divide $f(x) = 2x^3 + x^2 - 8x + 5$ by $x + 3$ using synthetic division.

$x+3=0$
$x=-3$

$$\begin{array}{r|rrrr} -3 & 2 & 1 & -8 & 5 \\ & & -6 & 15 & -21 \\ \hline & 2 & -5 & 7 & -16 \end{array}$$

$$\boxed{2x^2 - 5x + 7 + \frac{-16}{x+3}}$$

USING LONG DIVISION **Divide using polynomial long division.**

7. $(3x^3 + 11x^2 + 4x + 1) \div (x^2 + x)$

$$\boxed{3x + 8 + \frac{-4x+1}{x^2+x}}$$

$$x^2 + x + 0 \overline{\big)\, 3x^3 + 11x^2 + 4x + 1}$$

$$-\left(\cancel{3x^3} + 3x^2\right)$$

$$\cancel{8x^2} + 4x$$

$$-\left(\cancel{8x^2} + 8x\right)$$

$$-4x + 1$$

5.5 Apply the Remainder and Factor Theorems

EXAMPLE 4 Factor a polynomial

Factor the polynomial completely given that $x - 4$ is a factor.

$$f(x) = x^3 - 6x^2 + 5x + 12$$

$x - 4 = 0$
$x = 4$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 5 & 12 \\ & & 4 & -8 & -12 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

$(x-4)(x^2 - 2x - 3)$

$\boxed{(x-4)(x-3)(x+1)}$

EXAMPLE 4 **Factor a polynomial**

Factor $f(x) = 3x^3 - 4x^2 - 28x - 16$ completely given that $x + 2$ is a factor.

$x=-2$

-2	3	-4	-28	-16
		-6	20	16
	3	-10	-8	0

$(x+2)(3x^2 - 10x - 8)$

$AC = -24$

$-12 \quad 2$

$\underbrace{3x^2 - 12x}+\underbrace{2x - 8}$

$3x(x-4) + 2(x-4)$

$\boxed{(x+2)(x-4)(3x+2)}$

EXAMPLE 5

One zero of $f(x) = x^3 - 2x^2 - 23x + 60$ is $x = 3$. What is another zero of f?

$$\begin{array}{r|rrrr} 3 & 1 & -2 & -23 & 60 \\ & \downarrow & 3 & 3 & -60 \\ \hline & 1 & 1 & -20 & 0 \end{array}$$

$x^2 + x - 20 = 0$

$(x+5)(x-4) = 0$

$x = -5 \quad x = 4$

$\boxed{x = 3, 4, -5}$

EXAMPLE 5

Find the other zeros of f given that $f(-2) = 0$.

$$f(x) = x^3 + 2x^2 - 9x - 18$$

$x = -2$

FACTOR Given polynomial $f(x)$ and a factor of $f(x)$, factor $f(x)$ completely.

25. $f(x) = x^3 + 2x^2 - 51x + 108;\ x + 9$

$$
\begin{array}{r|rrrr}
-9 & 1 & 2 & -51 & 108 \\
 & & -9 & 63 & -108 \\
\hline
 & 1 & -7 & 12 & 0
\end{array}
$$

$(x+9)(x^2 - 7x + 12)$

$-4 \quad -3$

$\boxed{(x+9)(x-4)(x-3)}$

FIND ZEROS Given polynomial function f and a zero of f, find the other zeros.

29. $f(x) = x^3 - 2x^2 - 21x - 18; -3$

$$\begin{array}{r|rrrr} -3 & 1 & -2 & -21 & -18 \\ & & -3 & 15 & 18 \\ \hline & 1 & -5 & -6 & 0 \end{array}$$

$$x^2 - 5x - 6 = 0$$

$-6, 1$

$$(x-6)(x+1) = 0$$

$x = 6$ $\quad$ $x = -1$

$$\boxed{x = -3, -1, 6}$$