

5.6 Find Rational Zeros

EXAMPLE 1 List possible rational zeros

List the possible rational zeros of f using the rational zero theorem.

a. $f(x) = x^3 + 2x^2 - 11x + 12$

p - the factors of the constant term

$$p = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

q - the factors of the leading coeff.

$$q = \pm 1$$

all possible rational zeros are $\frac{p}{q}$.

$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

EXAMPLE 1 List possible rational zerosList the possible rational zeros of f using the rational zero theorem.

b. $f(x) = 4x^4 - x^3 - 3x^2 + 9x - 10$

$$p = \pm 1, \pm 2, \pm 5, \pm 10$$

$$q = \pm 1, \pm 2, \pm 4$$

$$\frac{p}{q} = \pm \frac{1}{1}, \pm \frac{2}{1}, \pm \frac{5}{1}, \pm \frac{10}{1}, \pm \frac{1}{2}, \pm \frac{2}{2}, \pm \frac{5}{2}, \pm \frac{10}{2}, \pm \frac{1}{4}, \pm \frac{2}{4}, \pm \frac{5}{4}, \pm \frac{10}{4}$$

$$\frac{p}{q} = \pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{4}, \pm \frac{5}{4}$$

EXAMPLE 1 List possible rational zeros

List the possible rational zeros of f using the rational zero theorem.

$$f(x) = 2x^3 + 3x^2 - 11x - 6$$

$$p = \pm 1, \pm 2, \pm 3, \pm 6$$

$$q = \pm 1, \pm 2$$

$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$$

EXAMPLE 2 Find zeros when the leading coefficient is 1Find all real zeros of $f(x) = x^3 - 8x^2 + 11x + 20$.

$$p = \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$$

$$q = \pm 1$$

$$\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$$

~~| | | | |
|---|----|----|----|
| 1 | -8 | 11 | 20 |
| | -1 | -7 | 4 |
| | -7 | 4 | 24 |~~

$$x = -1, 4, 5$$

-1	1	-8	11	20
	1	-9	20	0

$$x^2 - 9x + 20$$

$$\begin{matrix} & & -4 & -5 \\ & & \wedge & \wedge \\ (x-4)(x-5) & = & 0 \end{matrix}$$

$$x=4 \quad x=5$$

EXAMPLE 3 Find zeros when the leading coefficient is not 1

Find all real zeros of $f(x) = 10x^4 - 11x^3 - 42x^2 + 7x + 12$.

$p = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ $q = \pm 1, \pm 2, \pm 5, \pm 10$

$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{3}{5}, \pm \frac{4}{5}, \pm \frac{6}{5}, \pm \frac{12}{5}, \pm \frac{1}{10}, \pm \frac{3}{10}$

$-\frac{1}{2}$ 10 -11 -42 7 12

↓ ↗ ↗ ↗ ↗

10 -5 8 17 -12

10 -16 -34 24 0

$10x^3 - 16x^2 - 34x + 24$

$2(5x^3 - 8x^2 - 17x + 12)$

$\frac{3}{5}$ 5 -8 -17 12

↓ ↗ ↗ ↗

5 3 -3 -12

5 -5 -20 0

$5x^2 - 5x - 20$

$5(x^2 - x - 4)$

$ax^2 + bx + c$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-4)}}{2(1)}$

$x = -\frac{1}{2}, \frac{3}{5}, \frac{1+\sqrt{17}}{2}, \frac{1-\sqrt{17}}{2}$

$x = \frac{1 \pm \sqrt{17}}{2}$

ELIMINATING POSSIBLE ZEROS Use the graph to shorten the list of possible rational zeros of the function. Then find all real zeros of the function.

21. $f(x) = 6x^3 + 25x^2 + 16x - 15$

-3 $6 \quad 25 \quad 16 \quad -15$
 $\downarrow \quad \downarrow \quad \downarrow$
 $6 \quad 7 \quad -5 \quad 0$
 $6x^2 + 7x - 5 = 0$

AC = $\begin{matrix} -3 & 0 \\ 10 & -3 \end{matrix}$

$6x^2 + 10x - 3x - 5 = 0$

$2x(3x+5) - 1(3x+5) = 0$

$(3x+5)(2x-1) = 0$

$3x+5=0$

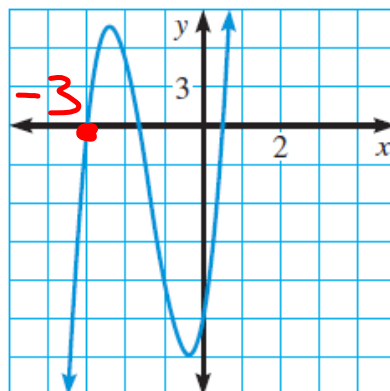
$2x-1=0$

$\frac{3x}{3} = \frac{-5}{3}$

$2x = 1$

$x = -\frac{5}{3}$

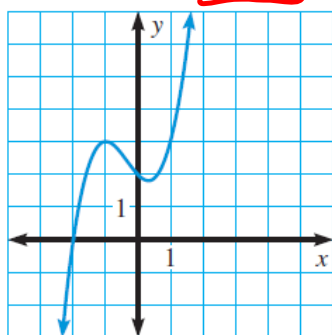
$x = \frac{1}{2}$



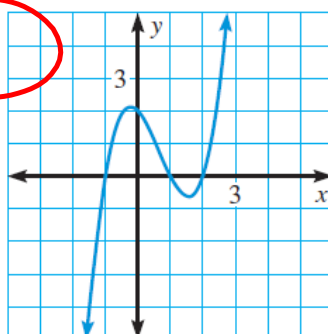
MATCHING Find all real zeros of the function. Then match each function with its graph.

$$41. f(x) = x^3 - 2x^2 - x + 2$$

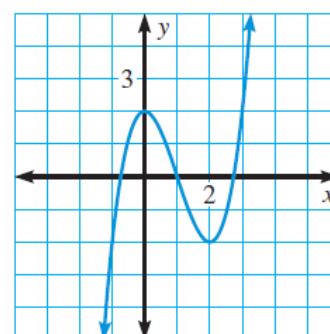
A.



B.



C.



$$\begin{aligned}
 &x^2(x-2) - 1(x-2) \\
 &(x-2)(x^2-1) \\
 &(x-2)(x-1)(x+1) \\
 &x=2 \quad x=1 \quad x=-1
 \end{aligned}$$