

5.7 Apply the Fundamental Theorem of Algebra

What is the Fundamental Theorem of Algebra?

Every polynomial has exactly the same # of solutions (zeros) as its degree. (may include i 's)

EXAMPLE 2 Find the zeros of a polynomial function

Find all zeros of the polynomial function.

$$f(x) = x^3 + 7x^2 + 15x + 9$$

Handwritten synthetic division work for the polynomial $f(x) = x^3 + 7x^2 + 15x + 9$.

Step 1: Synthetic division by $x + 1$ (using -1 as the divisor):

-1	1	7	15	9
	\downarrow	-1	-6	-9
	1	6	9	0

Step 2: Synthetic division by $x + 3$ (using -3 as the divisor):

-3	1	-3	-9
	\downarrow	-3	-9
	1	3	0

Equation: $x + 3 = 0$

Root: $x = -3$

$$x = -1, -3, -3$$

EXAMPLE 2 Find the zeros of a polynomial function

Find all zeros of $f(x) = x^5 - 4x^4 + 4x^3 + 10x^2 - 13x - 14$.

Handwritten synthetic division work:

Step 1: Dividing by 2

2	1	-4	4	10	-13	-14
	↓		2	-4	0	20
		↓			0	14
			1	-2	0	10
				↓		7
					1	-7
						0

Step 2: Dividing by -1

-1	1	-2	0	10	7	0
	↓		-1	3	-3	-7
		↓				0
			1	-3	3	7
				↓		0
					1	-7
						0

Step 3: Dividing by -1

-1	1	-3	3	7	0
	↓		-1	4	-7
		↓			0
			1	-4	7
				↓	
					0

Final quadratic: $x^2 - 4x + 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 28}}{2}$$

$$x = \frac{4 \pm \sqrt{-12}}{2}$$

$$x = \frac{4 \pm 2i\sqrt{3}}{2}$$

$$x = 2 \pm i\sqrt{3}$$

EXAMPLE 3 Use zeros to write a polynomial function

Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros.

$-1, 2, 4$

$4, 1 + \sqrt{5}, 1 - \sqrt{5}$

$3, 3 - i, 3 + i$

$$\begin{aligned} x &= -1 \\ x + 1 &= 0 \\ x &= 2 \\ x - 2 &= 0 \end{aligned}$$

$$f(x) = (x + 1)(x - 2)(x - 4)$$

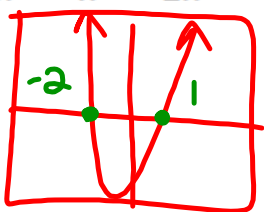
$$f(x) = (x - 4)(x - (1 + \sqrt{5}))(x - (1 - \sqrt{5}))$$

$$f(x) = (x - 3)(x - (3 - i))(x - (3 + i))$$

FINDING ZEROS Find all zeros of the polynomial function.

15. $f(x) = x^4 + x^3 + 2x^2 + 4x - 8$

graph



Handwritten synthetic division work:

$$\begin{array}{r|rrrrr} -2 & 1 & 1 & 2 & 4 & -8 \\ & & -2 & 2 & -8 & 8 \\ \hline & 1 & -1 & 4 & -4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & 4 & -4 & 0 \\ & & 1 & 0 & 4 & 4 \\ \hline & 1 & 0 & 4 & 0 & 4 \end{array}$$

$$x^2 + 0x + 4 = 0$$

$$x^2 + 4 = 0$$

Handwritten solution for the quadratic:

$$x^2 + 4 = 0$$

$$\sqrt{x^2} = \sqrt{-4}$$

$$x = \pm\sqrt{-4}$$

$x = \pm 2i$ (circled in green)

Final boxed answer:

$$x = -2, 1, -2i, 2i$$

WRITING POLYNOMIAL FUNCTIONS Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros.

24. $2, -i, i$