6.1 Evaluate *n*th Roots and Use Rational Exponents

Roots as Exponents

$\sqrt[2]{x} = x^{\frac{1}{2}}$

$\sqrt[3]{x} = x^{\frac{1}{3}}$

$\sqrt[5]{x} = x^{\frac{1}{5}}$

EXAMPLE 1 **Find *n*th roots**

Find the indicated real *n*th root(s) of *a*.

a. $n = 3, a = -216$

$\sqrt[3]{-216}$

$-6 \cdot 36$

$-6 \cdot -6$

$\sqrt[3]{(-6)^3} = (-6)^{\frac{3}{3}} = -6$

b. $n = 4, a = 81$

$\sqrt[4]{81}$

$9 \cdot 9$

$3 \cdot 3 \cdot 3 \cdot 3$

$\sqrt[4]{3^4} \quad 3^{\frac{4}{4}} = \pm 3$

Rational Exponents as Roots

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{or} \quad \left(\sqrt[n]{a}\right)^m$$

EXAMPLE 2 Evaluate expressions with rational exponents

Evaluate (a) $16^{3/2}$ and (b) $32^{-3/5}$.

a) $16^{\frac{3}{2}}$

$\sqrt{16^3}$ or $(\sqrt{16})^3$

$\sqrt{4096}$ $(\pm 4)^3$

± 64 ± 64

$32^{-\frac{3}{5}}$

$(\sqrt[5]{32})^{-3}$ $= (\sqrt[5]{2^5})^{-3}$

2 16

4 4 $= (2)^{-3}$

$= \frac{1}{2^3}$

$= \frac{1}{8}$

EXAMPLE 3 Approximate roots with a calculator

a. $9^{1/5}$ **b.** $12^{3/8}$ **c.** $(\sqrt[4]{7})^3$

$9 \wedge (1 \div 5)$

≈ 1.552

$12 \wedge (3 \div 8)$

≈ 2.539

$7^{\frac{3}{4}}$

$7 \wedge (3 \div 4)$

≈ 4.304

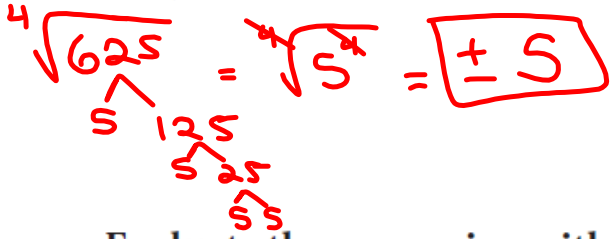
Mixed Review

Try these on your own in your notes:

Find the indicated real *n*th root(s) of *a*.

$n = 4, a = 625$

$n = 3, a = -64$



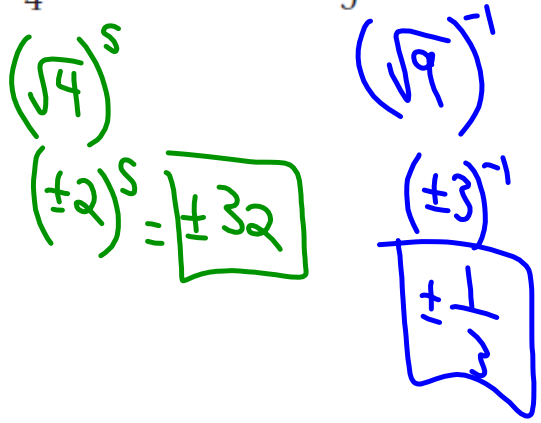
Evaluate the expression without using a calculator.

$4^{5/2}$

$9^{-1/2}$

$81^{3/4}$

$1^{7/8}$



EXAMPLE 4 Solve equations using *n*th roots

Solve the equation.

$4x^5 = 128$

$\frac{4x^5}{4} = \frac{128}{4}$

$\sqrt[5]{x^5} = \sqrt[5]{32}$

$x = 2$

$(x-3)^4 = 21$

$\sqrt[4]{(x-3)^4} = \sqrt[4]{21}$

$x - 3 = \pm\sqrt[4]{21}$

$x = 3 \pm \sqrt[4]{21}$