

6.1 Evaluate n th Roots and Use Rational Exponents

Roots as Exponents

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\sqrt[5]{x} = x^{\frac{1}{5}}$$

EXAMPLE 1 Find n th rootsFind the indicated real n th root(s) of a . $\sqrt[n]{a}$

a. $n = 3, a = -216$

$$\sqrt[3]{-216}$$

Handwritten tree diagram for $\sqrt[3]{-216}$:
-6 is circled and connected to 36.
36 is connected to -6 and -6, both of which are circled.

$$\sqrt[3]{(-6)^3} = (-6)^{\frac{3}{3}} = \boxed{-6}$$

b. $n = 4, a = 81$

$$\sqrt[4]{81}$$

Handwritten tree diagram for $\sqrt[4]{81}$:
81 is connected to 9 and 9.
Each 9 is connected to 3 and 3, both of which are circled.

$$\sqrt[4]{3^4} = 3^{\frac{4}{4}} = \boxed{\pm 3}$$

Rational Exponents as Roots

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{or} \quad (\sqrt[n]{a})^m$$

EXAMPLE 2 Evaluate expressions with rational exponentsEvaluate (a) $16^{3/2}$ and (b) $32^{-3/5}$.

a) $16^{3/2}$

$\sqrt{16^3}$

$\sqrt{4096}$

$\boxed{\pm 64}$

or $(\sqrt{16})^3$

$(\pm 4)^3$

$\boxed{\pm 64}$

$32^{-3/5}$

$(\sqrt[5]{32})^{-3}$

$\textcircled{2}$

$\textcircled{2} \textcircled{2} \textcircled{2} \textcircled{2} \textcircled{2}$

$(\sqrt[5]{2^5})^{-3}$

$= (2)^{-3}$

$\boxed{\frac{1}{8}}$

$2^3 = 8$

EXAMPLE 3 Approximate roots with a calculator

a. $9^{1/5}$

$$9 \wedge (1 \div 5)$$
$$\approx 1.552$$

b. $12^{3/8}$

$$12 \wedge (3 \div 8)$$
$$\approx 2.539$$

c. $(\sqrt[4]{7})^3$

$$7^{3/4}$$
$$7 \wedge (3 \div 4)$$
$$\approx 4.304$$

Mixed Review

Try these on your own in your notes:

Find the indicated real n th root(s) of a .

$$n = 4, a = 625 \qquad n = 3, a = -64$$

$$\sqrt[4]{625} = \sqrt[4]{5^4} = \boxed{\pm 5}$$

Evaluate the expression without using a calculator.

$$4^{5/2}$$

$$(\sqrt{4})^5$$

$$(\pm 2)^5 = \boxed{\pm 32}$$

$$9^{-1/2}$$

$$(\sqrt{9})^{-1}$$

$$(\pm 3)^{-1}$$

$$\boxed{\pm \frac{1}{3}}$$

$$81^{3/4}$$

$$1^{7/8}$$

EXAMPLE 4 Solve equations using n th roots

Solve the equation.

$$\begin{aligned} \frac{x^5}{4} &= \frac{128}{4} \\ \sqrt[5]{x} &= \sqrt[5]{32} \\ x &= 2 \end{aligned}$$

$$\begin{aligned} \sqrt[4]{(x-3)^4} &= \sqrt[4]{21} \\ x-3 &= \pm \sqrt[4]{21} \\ x &= 3 \pm \sqrt[4]{21} \end{aligned}$$