

6.2 Apply Properties of Rational Exponents

Exponent Properties

$$a^m \cdot a^n = a^{m+n}$$

$$(a^m)^n = a^{m \cdot n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(ab)^m = a^m b^m$$

$$2^{\frac{1}{2}} \cdot 2^{\frac{3}{2}} = 2^{\frac{1}{2} + \frac{3}{2}}$$

$$(2^{\frac{1}{2}})^6 = 2^{\frac{1}{2} \cdot 6} = 2^3$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$a^{-m} = \frac{1}{a^m}$$

EXAMPLE 1 Use properties of exponents

Use the properties of rational exponents to simplify the expression.

a. $7^{1/4} \cdot 7^{1/2}$

$$7^{1/4 + 1/2} = \boxed{7^{3/4}}$$

b. $(6^{1/2} \cdot 4^{1/3})^2$

$$(6^{1/2 \cdot 2} \cdot 4^{1/3 \cdot 2}) = \boxed{6^1 \cdot 4^{2/3}}$$

c. $(4^5 \cdot 3^5)^{-1/5}$

$$4^{5 \cdot (-1/5)} \cdot 3^{5 \cdot (-1/5)} = 4^{-1} \cdot 3^{-1} = (4 \cdot 3)^{-1} = 12^{-1} = \boxed{\frac{1}{12}}$$

d. $\frac{5^1}{5^{1/3}}$

$$5^{1 - 1/3} = \boxed{5^{2/3}}$$

e. $\left(\frac{42^{1/3}}{6^{1/3}}\right)^2$

$$\frac{42^{2/3}}{6^{2/3}} = \left(\frac{42}{6}\right)^{2/3} = \boxed{7^{2/3}}$$

Properties of Radicals

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

EXAMPLE 2 Use properties of radicals

a. $\sqrt[3]{12} \cdot \sqrt[3]{18}$

$$\begin{aligned} & \sqrt[3]{12 \cdot 18} \\ & \sqrt[3]{216} \\ & \begin{array}{l} \text{6} \quad \text{36} \\ \text{6} \quad \text{6} \end{array} \\ & \sqrt[3]{6^3} = \boxed{6} \end{aligned}$$

b. $\frac{\sqrt[4]{80}}{\sqrt[4]{5}}$

$$\begin{aligned} & \sqrt[4]{\frac{80}{5}} \\ & \sqrt[4]{16} = \sqrt[4]{2^4} = \boxed{\pm 2} \end{aligned}$$

EXAMPLE 3 Write radicals in simplest form

Write the expression in simplest form.

a. $\sqrt[3]{135}$

$$\begin{array}{l}
 \begin{array}{c}
 \swarrow \quad \searrow \\
 5 \quad 27 \\
 \swarrow \quad \searrow \\
 \quad 3 \quad 9 \\
 \quad \swarrow \quad \searrow \\
 \quad \quad 3 \quad 3
 \end{array} \\
 \sqrt[3]{5 \cdot 3^3} \\
 \sqrt[3]{5} \cdot \sqrt[3]{3^3} \\
 \boxed{\sqrt[3]{5}}
 \end{array}$$

b. $\frac{\sqrt[5]{7}}{\sqrt[5]{8}}$

$$\begin{aligned}
 &= \frac{7^{\frac{1}{5}}}{8^{\frac{1}{5}}} \\
 &= \frac{7^{\frac{1}{5}}}{(2^3)^{\frac{1}{5}}} = \frac{7^{\frac{1}{5}}}{2^{\frac{3}{5}}} \\
 &= \frac{7^{\frac{1}{5}}}{2^{\frac{3}{5}}} \cdot \frac{2^{\frac{2}{5}}}{2^{\frac{2}{5}}} = \frac{7^{\frac{1}{5}} \cdot 2^{\frac{2}{5}}}{2^1} \\
 &= \frac{\sqrt[5]{7 \cdot 2^2}}{2} = \boxed{\frac{\sqrt[5]{28}}{2}}
 \end{aligned}$$

PROPERTIES OF RATIONAL EXPONENTS Simplify the expression.

5. $3^{1/4} \cdot 27^{1/4}$

$$(3 \cdot 27)^{1/4}$$

$$81^{1/4}$$

$$\left(\cancel{81}^{1/4} \right)$$

$$\boxed{+3}$$

11. $\frac{120^{-2/5} \cdot 120^{2/5}}{7^{-3/4}}$

$$\frac{120^{-\frac{2}{5} + \frac{2}{5}}}{7^{-3/4}} = \frac{120^0}{7^{-3/4}}$$

$$\frac{1}{7^{-3/4}} = \boxed{7^{3/4}}$$

PROPERTIES OF RADICALS Simplify the expression.

21. $\frac{\sqrt[4]{36} \cdot \sqrt[4]{9}}{\sqrt[4]{4}}$

$$\sqrt[4]{\frac{36 \cdot 9}{4}} = \sqrt[4]{81} = \boxed{\pm 3}$$

SIMPLEST FORM Write the expression in simplest form.

$$\begin{aligned} 29. \frac{3}{\sqrt[4]{144}} &= \frac{3}{\sqrt[4]{2^4 \cdot 3^2}} = \frac{3}{2^{\frac{4}{4}} \cdot 3^{\frac{2}{4}}} \\ &= \frac{3^1}{2 \cdot 3^{\frac{1}{2}}} \\ &= \frac{3^{\frac{1}{2}}}{2} = \boxed{\frac{\sqrt{3}}{2}} \end{aligned}$$