

6.2 Apply Properties of Rational Exponents

EXAMPLE 5 Add and subtract like radicals and roots

Simplify the expression.

a. $\sqrt[4]{10} + 7\sqrt[4]{10}$

$x = \sqrt[4]{10}$

$x + 7x$

$8x$

$8\sqrt[4]{10}$

b. $2(8^{1/5}) + 10(8^{1/5})$

$(2 + 10)8^{1/5}$

$12(8^{1/5})$

EXAMPLE 5 Add and subtract like radicals and roots

Simplify the expression.

$$\begin{aligned} & \sqrt[3]{54} - \sqrt[3]{2} \\ & \begin{array}{c} 9 \quad 6 \\ \swarrow \quad \searrow \\ \textcircled{3} \textcircled{3} \textcircled{3} \textcircled{2} \end{array} \\ & \sqrt[3]{3^3 \cdot 2} - \sqrt[3]{2} \\ & \downarrow \\ & 3\sqrt[3]{2} - \sqrt[3]{2} \\ & (3-1)\sqrt[3]{2} \\ & \boxed{2\sqrt[3]{2}} \end{aligned}$$

$$\begin{aligned} & \sqrt[3]{5} + \sqrt[3]{40} \\ & \begin{array}{c} 2 \quad 20 \\ \swarrow \quad \searrow \\ \textcircled{2} \textcircled{2} \textcircled{2} \textcircled{2} \textcircled{5} \end{array} \\ & \sqrt[3]{5} + \sqrt[3]{2^3 \cdot 5} \\ & \downarrow \\ & \sqrt[3]{5} + 2\sqrt[3]{5} \\ & \boxed{3\sqrt[3]{5}} \end{aligned}$$

EXAMPLE 6 Simplify expressions involving variables

Simplify the expression. Assume all variables are positive.

a. $\sqrt[3]{64y^6}$

Handwritten solution: $\sqrt[3]{64y^6} = (2^6 y^6)^{1/3}$

Step 1: $(2^6 y^6)^{1/3}$

Step 2: $2^2 y^2$

Step 3: $4y^2$

b. $(27p^3q^{12})^{1/3}$

Handwritten solution: $(27p^3q^{12})^{1/3}$

Step 1: $27^{1/3} p^{3/3} q^{12/3}$

Step 2: $3 p^1 q^4$

Step 3: $3p^1q^4$

c. $\sqrt[4]{\frac{m^4}{n^8}}$

Handwritten solution: $\sqrt[4]{\frac{m^4}{n^8}} = \left(\frac{m^4}{n^8}\right)^{1/4}$

Step 1: $\frac{m^4}{n^8}$

Step 2: $\frac{m^4}{n^8}$

Step 3: $\frac{m}{n^2}$

d. $\frac{14xy^{1/3}}{2x^{3/4}z^{-6}}$

Handwritten solution: $\frac{14xy^{1/3}}{2x^{3/4}z^{-6}}$

Step 1: $7x^{1-3/4}y^{1/3}z^6$

Step 2: $7x^{1/4}y^{1/3}z^6$

EXAMPLE 7 Write variable expressions in simplest form

Write the expression in simplest form. Assume all variables are positive.

a. $\sqrt[5]{4a^8b^{14}c^5}$

$(4a^8b^{14}c^5)^{1/5}$

$4^{1/5} a^{8/5} b^{14/5} c^{5/5}$

$4^{1/5} \underbrace{a^1}_{a^1} \underbrace{a^{3/5}}_{a^{3/5}} \underbrace{b^2}_{b^2} \underbrace{b^{4/5}}_{b^{4/5}} c$

$ab^2c \left(4^{1/5} a^{3/5} b^{4/5} \right)$

$ab^2c \sqrt[5]{4a^3b^4}$

b. $\sqrt[3]{\frac{x}{y^8}}$

$\frac{x^{1/3}}{y^{8/3}} \cdot \frac{y^{1/3}}{y^{1/3}}$

$\frac{(xy)^{1/3}}{y^3} = \frac{\sqrt[3]{xy}}{y^3}$

EXAMPLE 8 Add and subtract expressions involving variables

Perform the indicated operation. Assume all variables are positive.

a. $\frac{1}{5}\sqrt{w} + \frac{3}{5}\sqrt{w}$

$$\left(\frac{1}{5} + \frac{3}{5}\right)\sqrt{w}$$

$$\frac{4}{5}\sqrt{w}$$

b. $3xy^{1/4} - 8xy^{1/4}$

$$(3 - 8)xy^{1/4}$$

$$-5xy^{1/4}$$

c. $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2}$

$$12\sqrt[3]{2z^3 \cdot z^2} - z\sqrt[3]{3^3 \cdot 2z^2}$$

$$12z\sqrt[3]{2z^2} - 3z\sqrt[3]{2z^2}$$

$$9z\sqrt[3]{2z^2}$$