

## 6.6 Solve Radical Equations

**EXAMPLE 5** Solve an equation with an extraneous solution

$$\begin{aligned}
 7+1 &= \sqrt{7(7)+15} \\
 8 &= \sqrt{49+15} \\
 8 &= \sqrt{64} \\
 8 &= 8
 \end{aligned}$$

$$\begin{aligned}
 -2+1 &= \sqrt{7(-2)+15} \\
 -1 &= \sqrt{-14+15} \\
 -1 &= \sqrt{1} \\
 -1 &= 1
 \end{aligned}$$

Solve  $x + 1 = \sqrt{7x + 15}$ .

$$\begin{aligned}
 (x+1)^2 &= (\sqrt{7x+15})^2 \\
 (x+1)(x+1) &= \\
 x^2 + x + x + 1 &= 7x + 15
 \end{aligned}$$

$$\begin{aligned}
 x^2 + 2x + 1 &= 7x + 15 \\
 -7x - 15 & \quad -7x - 15 \\
 x^2 - 5x - 14 &= 0
 \end{aligned}$$

$$\begin{aligned}
 (x-7)(x+2) &= 0 \\
 \boxed{x=7} \quad \cancel{x=-2}
 \end{aligned}$$

**EXAMPLE 5** Solve an equation with an extraneous solution

Solve the equation. Check for extraneous solutions.

$$\sqrt{10x+9} = (x+3)^2$$

$$\sqrt{10(0)+9} = 0+3$$

$$\sqrt{0+9} = 3$$

$$\sqrt{9} = 3$$

$$3 = 3$$


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$$\sqrt{10(4)+9} = 4+3$$

$$\sqrt{40+9} = 7$$

$$\sqrt{49} = 7$$

$$7 = 7$$

$$10x+9 = (x+3)(x+3)$$

$$10x+9 = x^2+3x+3x+9$$

$$10x+9 = x^2+6x+9$$

$$-10x-9 \quad -10-9$$

$$0 = x^2-4x$$

$$0 = x(x-4)$$

$$x=0 \quad x-4=0$$

$$x=4$$

**EXAMPLE 6** Solve an equation with two radicals

$\sqrt{2+2} + 1 = \sqrt{3-2}$   
 $\sqrt{4} + 1 = \sqrt{1}$   
 $2 + 1 = 1$   
 ~~$3 = 1$~~

$\sqrt{-1+2} + 1 = \sqrt{3-1}$   
 $\sqrt{1} + 1 = \sqrt{2}$   
 $1 + 1 = 2$   
 $2 = 2$

Solve  $(\sqrt{x+2} + 1)^2 = (\sqrt{3-x})^2$   
 $(\sqrt{x+2} + 1)(\sqrt{x+2} + 1) = 3 - x$   
 $(x+2) + \sqrt{x+2} + \sqrt{x+2} + 1 = 3 - x$   
 ~~$x + 3$~~  +  $2\sqrt{x+2} = 3 - x$   
 ~~$-x - 3$~~        ~~$-3 - x$~~

$2\sqrt{x+2} = -2x$   
 $\sqrt{x+2} = (-x)$   
 $x+2 = x^2$

$0 = x^2 - x - 2$   
 $(x-2)(x+1)$   
 ~~$x=2$~~        $x=-1$

**EXAMPLE 6** Solve an equation with two radicals

Solve the equation. Check for extraneous solutions.

$$(\sqrt{x+6} - 2)^2 = \sqrt{x-2}$$

$$(\sqrt{x+6} - 2)(\sqrt{x+6} - 2) = x - 2$$

$$(x+6) - 2\sqrt{x+6} - 2\sqrt{x+6} + 4 = x - 2$$

$$x+6+4 - 4\sqrt{x+6} = x-2$$

$$x+10 - 4\sqrt{x+6} = x-2$$

$$\frac{-4\sqrt{x+6} = -8}{-4} \quad \frac{-8}{-4}$$

$$\sqrt{x+6} = 2$$

$$x+6 = 4$$

$$x = -2$$

no solution

$$\sqrt{-2+6} - 2 = \sqrt{-2-2}$$

$$\sqrt{4} - 2 = \sqrt{-4}$$

$$2 - 2 = \sqrt{-4}$$

$$0 = 2$$