7.5 Apply Properties of Logarithms

Properties of Logarithms

Product Property- $\log_b(mn) = \log_b m + \log_b n$

Quotient Property- $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

Power Property- $\log_b m^n = n \log_b m$

EXAMPLE 1 Use properties of logarithms

Use $\log_4 3 \approx 0.792$ and $\log_4 7 \approx 1.404$ to evaluate the logarithm.

$$\log_4 \frac{3}{7}$$

$\log_4 3 - \log_4 7$

$(0.792) - (1.404)$

$\boxed{-0.612}$

EXAMPLE 1 Use properties of logarithms

Use $\log_4 3 \approx 0.792$ and $\log_4 7 \approx 1.404$ to evaluate the logarithm.

$\log_4 21$

$\log_4(3 \cdot 7)$

$\log_4 3 + \log_4 7$

$(0.792) + (1.404)$

2.196

$\log_4 49$

$\log_4 7^2$

$2\log_4 7$

$2(1.404)$

2.808

EXAMPLE 2 **Expand a logarithmic expression**

Expand $\log_6 \frac{5x^3}{y}$.

$\log_6 5x^3 - \log_6 y$

$\log_6 5 + \log_6 x^3 - \log_6 y$

$\log_6 5 + 3\log_6 x - \log_6 y$

EXAMPLE 2 **Expand a logarithmic expression**

Expand $\log 3x^4$

$\log 3 + \log x^4$

$\log 3 + 4\log x$

APPROXIMATING EXPRESSIONS Use $\log 4 \approx 0.602$ and $\log 12 \approx 1.079$ to evaluate the logarithm.

9. $\log 16$

$\log 4^{(2)}$

$2 \log 4$

$2(0.602)$

1.204

13. $\log \frac{1}{4}$

$\log (4^{-1})$

$-\log 4$

-0.602

EXPANDING EXPRESSIONS Expand the expression.

19. $\log_2 \frac{2}{5}$

$\log_2 2 - \log_2 5$

$1 - \log_2 5$

23. $\log_7 5x^3yz^2$

$\log_7 5 + \log_7 x^3 + \log_7 y + \log_7 z^2$

$\log_7 5 + 3\log_7 x + \log_7 y + 2\log_7 z$

EXPANDING EXPRESSIONS **Expand the expression.**

27. $\log_2 \sqrt{x}$

$\log_2 x^{1/2}$

$\frac{1}{2}\log_2 x$

7.5 Apply Properties of Logarithms

Properties of Logarithms

Product Property- $\log_b(m) + \log_b(n) = \log_b(m \cdot n)$

Quotient Property- $\log_b(m) - \log_b(n) = \log_b\left(\frac{m}{n}\right)$

Power Property- $n\log_b(m) = \log_b(m^n)$

EXAMPLE 3

Which of the following is equivalent to $\log 9 + 3 \log 2 - \log 3$?

(A) $\log 8$ (B) $\log 14$ (C) $\log 18$ (D) $\log 24$

$$\log 9 + 3\log 2 - \log 3$$

$$\log 9 + \log 2^3 - \log 3$$

$$\log\left(\frac{9 \cdot 2^3}{3}\right)$$

$$\log(3 \cdot 8)$$

$$\log 24$$

EXAMPLE 3

Condense $\ln 4 + 3 \ln 3 - \ln 12$

$\ln 4 + \ln 3^3 - \ln 12$

$\ln\left(\frac{4 \cdot 3^3}{12_3}\right)$

$\ln\left(\frac{3^{3_2}}{3}\right) = \ln(3^2) = \boxed{\ln 9}$

Change of Base Formula

$$\log_b A = \frac{\log_c A}{\log_c b} = \frac{\log A}{\log b} = \frac{\ln A}{\ln b}$$

EXAMPLE 4 **Use the change-of-base formula**

Evaluate $\log_3 8$ using common logarithms and natural logarithms.

$$\log_3 8 = \frac{\log 8}{\log 3} = \frac{0.9031}{0.4771} \approx 1.893$$

$$\frac{\ln 8}{\ln 3} = \frac{2.079}{1.097} \approx 1.893$$

EXAMPLE 4 **Use the change-of-base formula**

Use the change-of-base formula to evaluate the logarithm.

$\log_8 14$

$\frac{\log 14}{\log 8} \approx 1.269$

$\log_{26} 9$

$\frac{\ln 9}{\ln 26} \approx 0.674$

CONDENSING EXPRESSIONS **Condense the expression.**

35. $2 \log x + \log 11$

$\log(x^2) + \log(11)$

$\log(11x^2)$

39. $\ln 40 + 2 \ln \frac{1}{2} + \ln x$

$\ln 40 + \ln\left(\frac{1}{2}\right)^2 + \ln x$

$\ln 40 + \ln \frac{1}{4} + \ln x$

$\ln\left(40 \cdot \frac{1}{4} \cdot x\right)$

$\ln(10x)$

CHANGE-OF-BASE FORMULA Use the change-of-base formula to evaluate the logarithm.

45. $\log_4 7$ $= \frac{\log 7}{\log 4}$ or $\frac{\ln 7}{\ln 4}$

≈ 1.404