

7.6 Solve Exponential and Logarithmic Equations

EXAMPLE 1 Solve by equating exponents

Solve the equation.

$$9^{2x} = 27^{x-1}$$

$$(3^2)^{2x} = (3^3)^{x-1}$$

$$3^{4x} = 3^{3x-3}$$

$$4x = 3x - 3$$

$$x = -3$$

EXAMPLE 1 Solve by equating exponents

Solve $4^x = \left(\frac{1}{2}\right)^{x-3}$.

$$(2^2)^x = (2^{-1})^{x-3}$$

$$2x = -x + 3$$
$$2 = 2$$

$$2x = -x + 3$$
$$+x \quad +x$$

$$3x = 3$$

$$x = 1$$

EXAMPLE 1 Solve by equating exponents

Solve the equation.

$$100^{7x+1} = 1000^{3x-2}$$

$$(10^2)^{7x+1} = (10^3)^{3x-2}$$

$$10^{14x+2} = 10^{9x-6}$$

$$\begin{array}{r} 14x+2 = 9x-6 \\ -9x \quad -9x \\ \hline 5x+2 = -6 \end{array}$$

$$\begin{array}{r} 5x+2 = -6 \\ -2 \quad -6 \\ \hline 5x = -8 \end{array}$$

$$\begin{array}{r} 5x = -8 \\ \hline x = -\frac{8}{5} \end{array}$$

$$\boxed{x = -\frac{8}{5}}$$

$$81^{3-x} = \left(\frac{1}{3}\right)^{5x-6}$$

$$(3^4)^{3-x} = (3^{-1})^{5x-6}$$

$$3^{12-4x} = 3^{-5x+6}$$

$$12-4x = -5x+6$$

$$\begin{array}{r} 12-4x = -5x+6 \\ +5x \quad +5x \\ \hline 12+x = 6 \end{array}$$

$$\begin{array}{r} 12+x = 6 \\ -12 \quad -12 \\ \hline x = -6 \end{array}$$

$$\boxed{x = -6}$$

EXAMPLE 2 Take a logarithm of each sideSolve $4^x = 11$.

$$B^P = A$$
$$\log_B A = P$$

$$\log_4 11 = x$$

$$x = \frac{\log(11)}{\log(4)} \approx 1.730$$

$$4^x = 11$$

$$\log(4^x) = \log(11)$$

$$\frac{x \log 4}{\log 4} = \frac{\log 11}{\log 4}$$

$$x = \frac{\log 11}{\log 4} \approx 1.730$$

EXAMPLE 2 Take a logarithm of each side

Solve the equation.

$7^{9x} = 15$

$$\log(7^{9x}) = \log 15$$

$$\cancel{9} x \log \cancel{7} = \frac{\log 15}{\cancel{9} \log \cancel{7}}$$

$$x = \frac{\log(15)}{(9 \log(7))} \approx 0.155$$

$4e^{-0.3x} - 7 = 13$
 $+7 +7$

$$\cancel{4} e^{-0.3x} = \frac{20}{\cancel{4}}$$

$$\ln(\cancel{e}^{-0.3x}) = \ln(5)$$

$$\cancel{0.3} x = \frac{\ln 5}{\cancel{-0.3} \cancel{0.3}}$$

$$x \approx -5.365$$