

7.6 Solve Exponential and Logarithmic Equations

EXAMPLE 4 Solve a logarithmic equation

Solve $\log_5(4x - 7) = \log_5(x + 5)$.

$$\begin{aligned} \log_5(4(4) - 7) &= \log_5(4 + 5) \\ \log_5 9 &= \log_5 9 \end{aligned}$$
$$\begin{aligned} 4x - 7 &= x + 5 \\ -x & \quad -x \\ 3x - 7 &= 5 \\ +7 & \quad +7 \\ 3x &= 12 \\ \frac{3}{3} & \quad \frac{3}{3} \\ x &= 4 \end{aligned}$$

EXAMPLE 4 Solve a logarithmic equation

Solve the equation. Check for extraneous solutions.

$$\ln(7x - 4) = \ln(2x + 11)$$

$$\begin{array}{r} 7x - 4 = 2x + 11 \\ -2x \quad -2x \end{array}$$

$$\ln(7(3) - 4) = \ln(2(3) + 11) \quad \begin{array}{r} 5x - 4 = 11 \\ +4 \quad +4 \end{array}$$

$$\ln(17) = \ln(17)$$

$$\begin{array}{r} 5x = 15 \\ \frac{5}{5} \quad \frac{5}{5} \end{array}$$

$$\boxed{x = 3}$$

EXAMPLE 5 Exponentiate each side of an equationSolve $\log_4(5x - 1) = 3$.

$$\log_B A = P$$

$$B^P = A$$

$$4^3 = 5x - 1$$

$$64 = 5x - 1$$

$$\frac{65}{5} = \frac{5x}{5}$$

$$x = 13$$

$$\log_4(5(13) - 1) = 3$$

$$\log_4 64 = 3$$

~~$$\log_4(5x - 1) = 4^3$$~~

$$5x - 1 = 4^3$$

$$x = 13$$

EXAMPLE 5 Exponentiate each side of an equation

Solve the equation. Check for extraneous solutions.

$$\log_2(x - 6) = 5$$

$$\cancel{2} \quad 2$$

$$x - 6 = 2^5$$

$$x - 6 = 32$$

$$\boxed{x = 38}$$

$$\log_2(38 - 6) = 5$$

$$\log_2(32) = 5$$

EXAMPLE 6

What is (are) the solution(s) of $\log 2x + \log (x - 5) = 2$?

$$\log 2x + \log (x - 5) = 2$$

~~$$\log (2x(x - 5)) = 2$$~~

$$2x(x - 5) = 10^2$$

$$2x^2 - 10x = 100$$

$$\frac{2x^2}{2} - \frac{10x}{2} - \frac{100}{2} = 0$$

$$x^2 - 5x - 50 = 0$$

$$(x - 10)(x + 5) = 0$$

$$x = 10$$

~~$$x = -5$$~~

~~$$\log(2(10)) + \log(10 - 5)$$~~
~~$$\log(20) + \log(5)$$~~
~~$$\log(2(-5)) + \log(-5 - 5)$$~~
~~$$\log(-10) + \log(-10)$$~~

EXAMPLE 6

Solve the equation. Check for extraneous solutions.

$$\log_4(x+12) + \log_4 x = 3$$

~~$$\log_4((x+12)(x)) = 4^3$$~~

$$(x+12)(x) = 4^3$$

$$x^2 + 12x = 64$$

-64 -64

$$x^2 + 12x - 64 = 0$$

$$(x+16)(x-4) = 0$$

~~$x = -16$~~ $x = 4$

SOLVING LOGARITHMIC EQUATIONS Solve the equation. Check for extraneous solutions.

27. ~~$\log_5(2x - 7) = \log_5(3x - 9)$~~

~~$2x - 7 = 3x - 9$~~
 ~~$-2x + 9 \quad -2x + 9$~~

~~$2 = x$~~

$\log_5(2(2) - 7) = \log_5(3(2) - 9)$

~~$\log_5(-3) = \log_5(-3)$~~

no solution

EXPONENTIATING TO SOLVE EQUATIONS Solve the equation. Check for extraneous solutions.

$$41. \log_5(x+4) + \log_5(x+1) = 2$$

$$\cancel{5} \log_5((x+4)(x+1)) = \cancel{5} \cdot 2$$

$$(x+4)(x+1) = 5^2$$

$$x^2 + x + 4x + 4 = 25$$

$$x^2 + 5x + 4 = 25$$

$$-25 - 25$$

$$x^2 + 5x - 21 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4(1)(-21)}}{2(1)}$$

$$\frac{-5 \pm \sqrt{109}}{2}$$

↑

$$x = \frac{-5 \pm \sqrt{109}}{2}$$