8.1 Model Inverse and Joint Variation

Direct Variation	*Inverse Variation*
$y = ax$	$y = \frac{a}{x}$
Find the equation of Variation when $x = 3$ and $y = -6$	$3 \cdot -6 = \frac{a}{3} \cdot 3$
$\frac{-6}{3} = \frac{a}{3}$ $a = -2$	$-18 = a$
$y = -2x$	$y = \frac{-18}{x}$

EXAMPLE 1 **Classify direct and inverse variation**

Tell whether x and y show *direct variation*, *inverse variation*, or *neither*.

Given Equation	Rewritten Equation	Type of Variation
a. $xy = 7$	$y = \frac{7}{x}$	$y = \frac{a}{x}$ Inverse
$y = ax + b$ b. $y = x + 3$		neither
c. $\frac{y}{4} = x$	$y = 4x$	$y = ax$ direct

EXAMPLE 2 **Write an inverse variation equation**

The variables x and y vary inversely, and $y = 7$ when $x = 4$. Write an equation that relates x and y. Then find y when $x = -2$.

$y = \frac{a}{x}$

$4 \cdot 7 = \frac{a}{4} \cdot 4$

$a = 28$

$y = \frac{28}{x}$

$y = \frac{28}{-2}$

$y = -14$

EXAMPLE 2 **Write an inverse variation equation**

The variables x and y vary inversely. Use the given values to write an equation relating x and y. Then find y when $x = 2$.

$x = 8, y = -1$

$y = \frac{a}{x}$

$8 \cdot -1 = \frac{a}{8} \cdot 8$

$a = -8$

$y = \frac{-8}{x}$

$y = \frac{-8}{2}$

$y = -4$

$x = \frac{1}{2}, y = 12$

$y = \frac{a}{x}$

$\frac{1}{2} \cdot 12 = \frac{a}{\frac{1}{2}} \cdot \frac{1}{2}$

$6 = a$

$y = \frac{6}{x}$

$y = \frac{6}{2}$

$y = 3$

Joint Variation

$$z = axy$$

or

$$z = axywkvm$$

EXAMPLE 5 **Write a joint variation equation**

The variable z varies jointly with x and y. Also, $z = -75$ when $x = 3$ and $y = -5$. Write an equation that relates x, y, and z. Then find z when $x = 2$ and $y = 6$.

$z = axy$

$-75 = a(3)(-5)$

$\frac{-75}{-15} = \frac{-15a}{-15}$

$a = 5$

$z = 5xy$

$z = 5(2)(6)$

$z = 60$

EXAMPLE 5 **Write a joint variation equation**

The variable z varies jointly with x and y. Use the given values to write an equation relating x, y, and z. Then find z when $x = -2$ and $y = 5$.

$x = 1, y = 2, z = 7$

$z = axy$

$7 = a(1)(2)$

$\frac{7}{2} = \frac{2a}{2}$

$a = \frac{7}{2}$

$z = \frac{7}{2}xy$

$z = \frac{7}{2}(-2)(5)$

$z = -35$

$x = 4, y = -3, z = 24$

$z = axy$

$24 = a(4)(-3)$

$\frac{24}{-12} = \frac{-12a}{-12}$

$a = -2$

$z = -2xy$

$z = -2(-2)(5)$

$z = 20$

EXAMPLE 6 Compare different types of variation

Write an equation for the given relationship.

Relationship	Equation
a. y varies inversely with x.	$y = \frac{a}{x}$
b. z varies jointly with x, y, and r.	$z = axyr$
c. y varies inversely with the square of x.	$y = \frac{a}{x^2}$
d. z varies directly with y and inversely with x.	$z = \frac{ay}{x}$
e. x varies jointly with t and r and inversely with s.	$x = \frac{atr}{s}$

PROBLEM SOLVING

37. **DIGITAL CAMERAS** The number n of photos your digital camera can store varies inversely with the average size s (in megapixels) of the photos. Your digital camera can store 54 photos when the average photo size is 1.92 megapixels. Write a model that gives n as a function of s. How many photos can your camera store when the average photo size is 3.87 megapixels?

$n = \frac{q}{s}$

$(1.92)54 = \frac{q}{1.92}(1.92)$

$q = 103.68$

$n = \frac{103.68}{s}$

$n = \frac{103.68}{3.87}$

$n = 26.791\ldots$

$n = 26$ pics