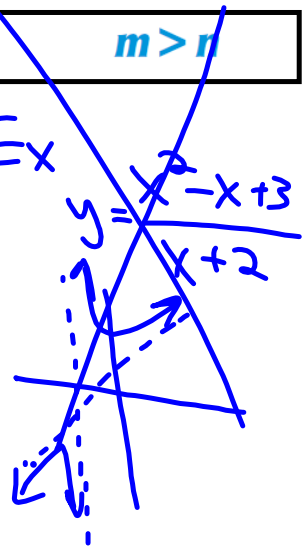
8.3 Graph General Rational Functions

$$f(x) = \frac{p(x)}{q(x)} = \frac{a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0}{b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0}$$

deg deg

Three Degree Situations:

$m < n$	$m = n$	$m > n$
Ex: $y = \frac{x}{x^2+1}$ H.A. $y=0$	Ex $y = \frac{3x+5}{2x+1}$ HA. $y = \frac{l.c.}{l.c.}$ $y = \frac{3}{2}$	Ex $y = \frac{x^2-x+3}{x+2}$ 

EXAMPLE 1 **Graph a rational function ($m < n$)**

Graph the function.

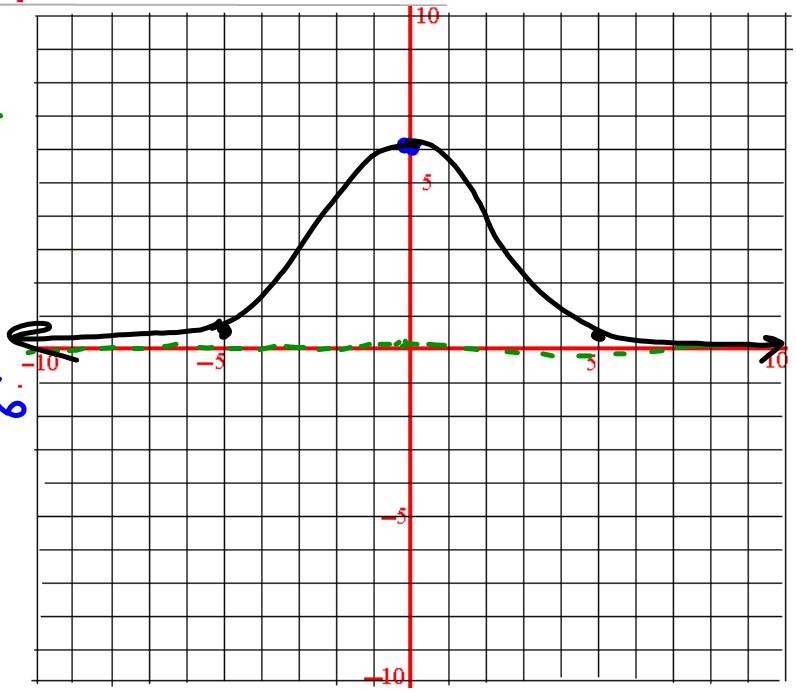
$$y = \frac{6}{x^2 + 1}$$

deg = 0
deg = 2

H.A. $y = 0$

x-int: $6 = 0$ → none

y-int: $\frac{6}{0^2+1} = 6$ → (0, 6)



Vertical Asymptote(s): none

Horizontal Asymptote: $y = 0$

X-int: none

Y-int: (0, 6) Domain: $\mathbb{R}$

EXAMPLE 2 **Graph a rational function ($m = n$)**

Graph the function.

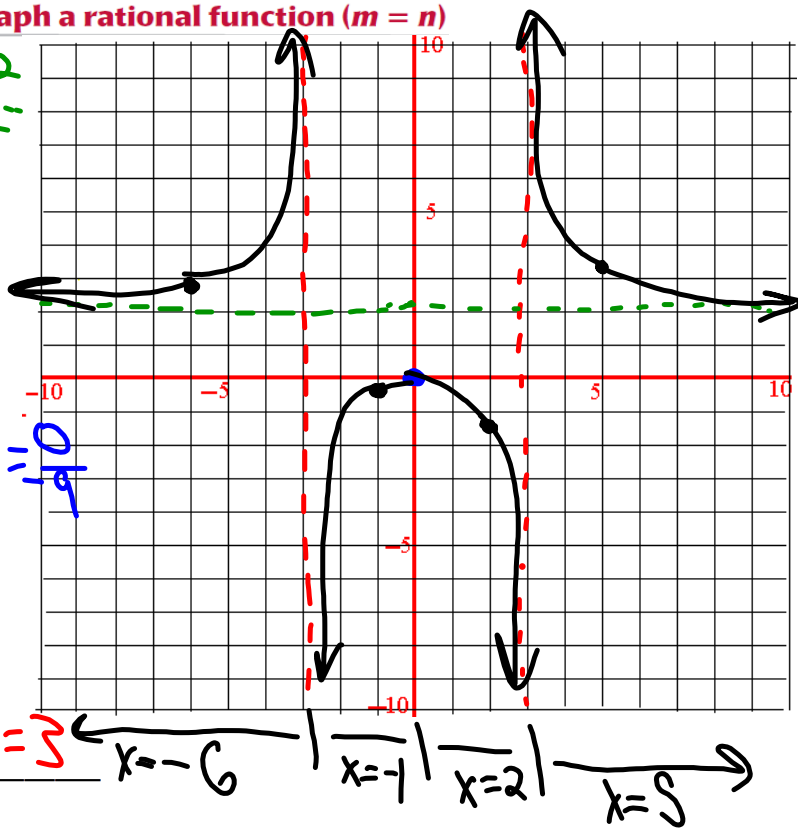
$$y = \frac{2x^2}{x^2 - 9}$$

deg = 2
deg = 2

H.A.
$y = \frac{2}{1}$ $y = 2$

x-int:
$2x^2 = 0$
$x^2 = 0$
$x = 0$
(0,0)

y-int:
$\frac{2(0)^2}{0^2 - 9} = \frac{0}{-9}$
$= 0$
(0,0)



$x = -3$, $x = 3$

$x = -6$ | $x = -1$ | $x = 2$ | $x = 5$

Vertical Asymptote(s): $x = -3$, $x = 3$

Horizontal Asymptote: $y = 2$

X-int: (0,0)

Y-int: (0,0) Domain: $\mathbb{R}, x \neq -3, 3$

Graph the function.

$$y = \frac{3x^2}{x^2 - 1}$$

V.A. $x^2 - 1 = 0$

$x = \pm 1$

H.A. $y = \frac{3}{1} = 3$

x-int:

$3x^2 = 0$

$x^2 = 0$

$x = 0$

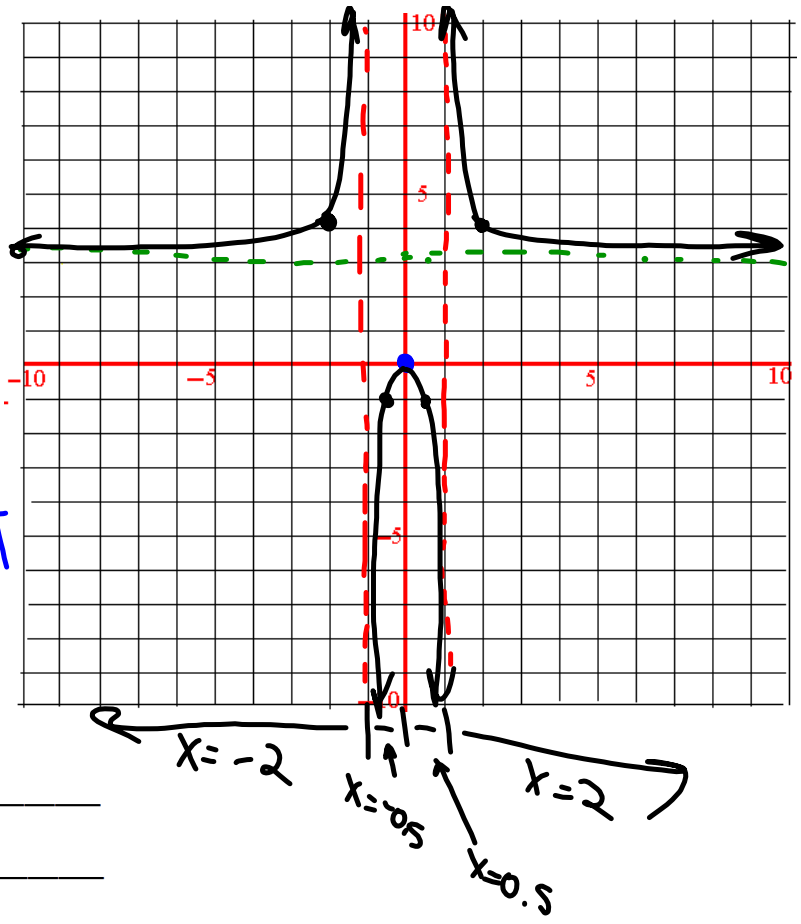
(0,0)

y-int:

$\frac{3(0)^2}{(0)^2 - 1} = \frac{0}{-1}$

$= 0$

(0,0)



Vertical Asymptote(s): $x = 1$, $x = -1$

Horizontal Asymptote: $y = 3$

X-int: (0,0)

Y-int: (0,0) Domain: $\mathbb{R}, x \neq \pm 1$

Graph the function.

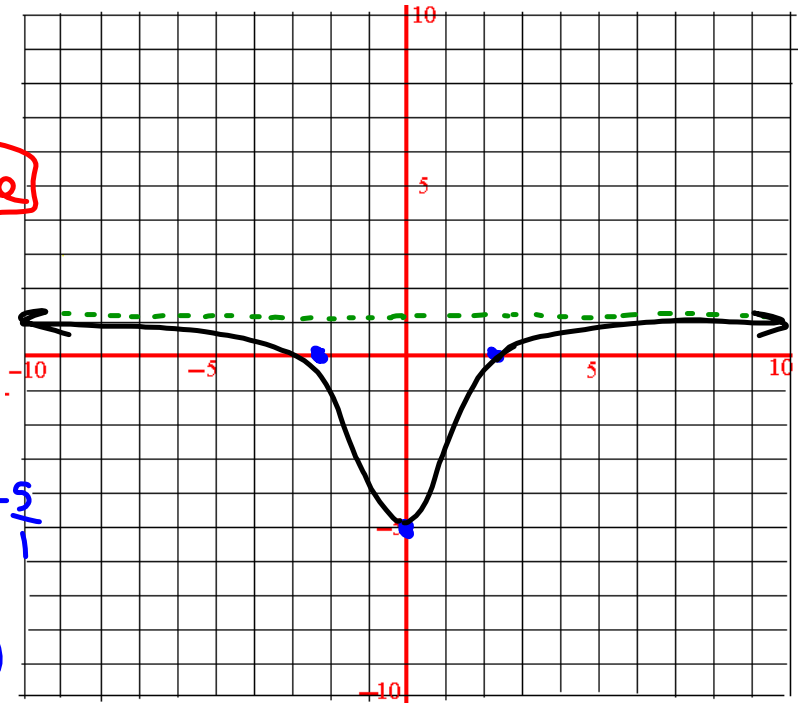
$$f(x) = \frac{x^2 - 5}{x^2 + 1}$$

VA $x^2+1=0$ $x^2=-1$ none

H.A $y=\frac{1}{1} \rightarrow y=1$

x-int: $x^2-5=0$, $x^2=5$, $x=\pm\sqrt{5}$, $(\sqrt{5},0),(-\sqrt{5},0)$

y-int: $\frac{0^2-5}{0^2+1}=\frac{-5}{1}=-5$, $(0,-5)$



Vertical Asymptote(s): none

Horizontal Asymptote: $y=1$

X-int: $(\sqrt{5},0),(-\sqrt{5},0)$

Y-int: $(0,-5)$ Domain: $\mathbb{R}$

MATCHING GRAPHS Match the function with its graph.

3. $y = \dfrac{-10}{x^2 - 9}$ → V.A.

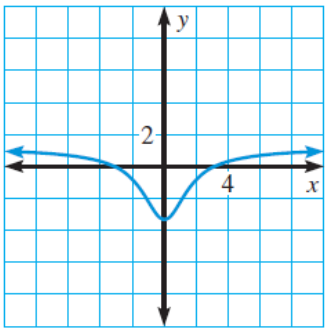
$x^2 - 9 = 0$

$x^2 = 9$

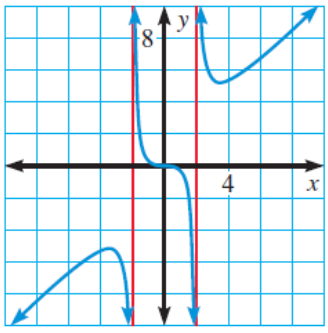
$x = \pm 3$

V.A. @ $x = 3$ & $x = -3$

A.



B.



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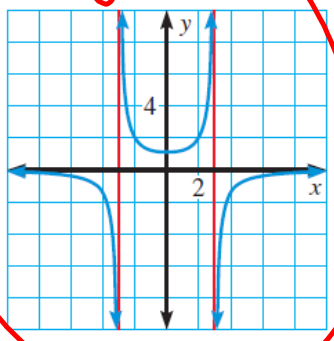


ANALYZING GRAPHS Identify the x-intercept(s) and vertical asymptote(s) of the graph of the function.

7. $y = \frac{5}{x^2 - 1}$

X-int:

$5 = 0$

none

V.A.

$x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

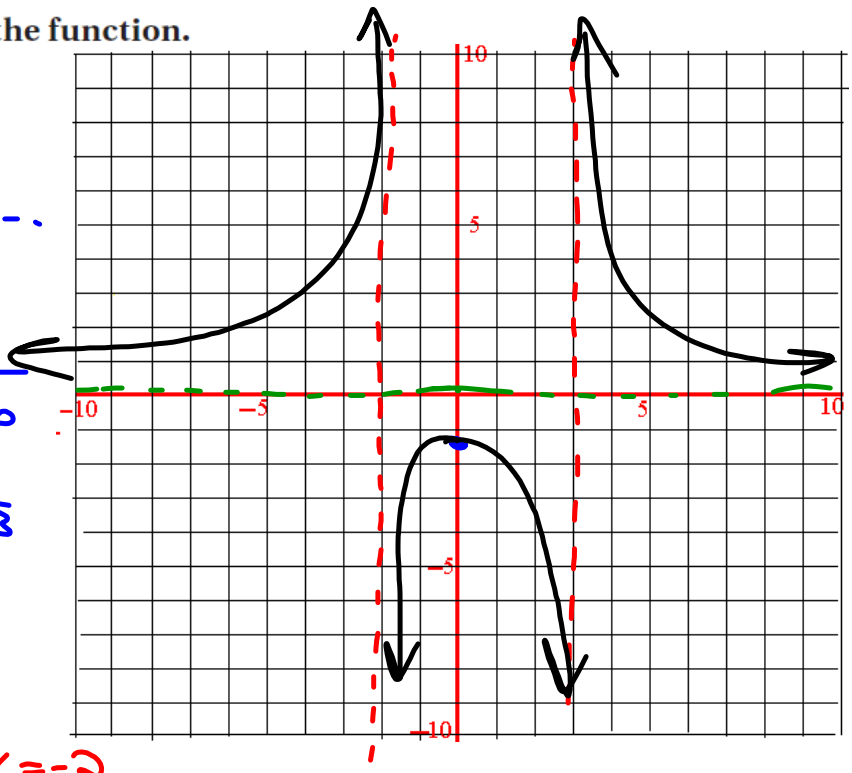
GRAPHING FUNCTIONS Graph the function.

16. $y = \frac{8}{x^2 - x - 6}$

H.A. $y = 0$

x-int: $8 = 0$ none

y-int: $\frac{8}{0^2 - 0 - 6} = \frac{8}{-6} = -\frac{4}{3}$ $\left(0, -\frac{4}{3}\right)$



Vertical Asymptote(s): $x = 3$, $x = -2$

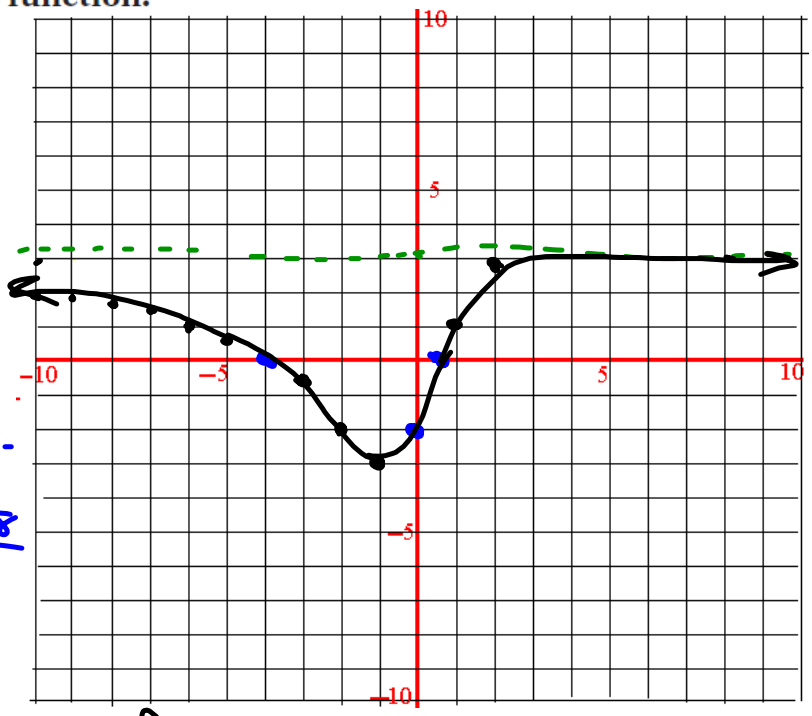
Horizontal Asymptote: $y = 0$

X-int: none

Y-int: $(0, -\frac{4}{3})$ Domain: $\mathbb{R}, x \neq 3, -2$

GRAPHING FUNCTIONS Graph the function.

23. $h(x) = \dfrac{3x^2 + 10x - 8}{x^2 + 4}$



V.A. $x^2+4=0$

none

H.A. $y = \frac{3}{1}$ $y=3$

X-int: $3x^2+10x-8=0$

$(3x-2)(x+4)=0$

$x=\frac{2}{3}$ $x=-4$

y-int: $\frac{3(0)^2+10(0)-8}{(0)^2+4}$

$=\frac{-8}{4}=-2$

none

Vertical Asymptote(s): none

Horizontal Asymptote: $y=3$

X-int: $(\frac{2}{3},0)$, $(-4,0)$

Y-int: $(0,-2)$ Domain:

Plug in points