

8.5 Add and Subtract Rational Expressions

$$\text{Add } \frac{7}{20} + \frac{9}{20} = \frac{7+9}{20} = \frac{16}{20} = \frac{\cancel{4} \cdot 4}{\cancel{4} \cdot 5} = \boxed{\frac{4}{5}}$$

Find the least common multiple of 20 and 45.

$$\begin{array}{l} 5 \cdot 2^2 \cdot 3^2 \\ 5 \cdot 4 \cdot 9 \\ 20 \cdot 9 \\ \hline 180 \end{array}$$

$$\begin{array}{l} 20 \\ \swarrow \searrow \\ 10 \quad 2 \\ \swarrow \searrow \\ 5 \quad 2 \\ \hline 5 \cdot 2^2 \end{array} \qquad \begin{array}{l} 45 \\ \swarrow \searrow \\ 9 \quad 5 \\ \swarrow \searrow \\ 3 \quad 3 \\ \hline 5 \cdot 3^2 \end{array}$$

$$\begin{array}{l} \text{Add } \frac{5}{18} + \frac{31}{72} \longrightarrow \\ \frac{4 \cdot 5}{4 \cdot 18} + \frac{31}{72} \\ \frac{20}{72} + \frac{31}{72} = \frac{51}{72} = \frac{\cancel{3} \cdot 17}{\cancel{3} \cdot 24} = \boxed{\frac{17}{24}} \end{array}$$

EXAMPLE 1 Add or subtract with like denominators

Perform the indicated operation.

a. $\frac{7}{4x} + \frac{3}{4x}$

$$\frac{7+3}{4x}$$

$$\frac{10}{4x} = \frac{5}{2x}$$

b. $\frac{2x}{x+6} - \frac{5}{x+6}$

$$\frac{2x-5}{x+6}$$

EXAMPLE 2 Find a least common multiple (LCM)Find the least common multiple of $4x^2 - 16$ and $6x^2 - 24x + 24$.

$$\begin{aligned}4x^2 - 16 \\4(x^2 - 4) \\4(x-2)(x+2) \\2^2(x-2)(x+2)\end{aligned}$$

$$\begin{aligned}6x^2 - 24x + 24 \\6(x^2 - 4x + 4) \\6(x-2)(x-2) \\6(x-2)^2 \\2 \cdot 3(x-2)^2\end{aligned}$$

$$3 \cdot 2^2 (x-2)^2 (x+2)$$

EXAMPLE 3 Add with unlike denominators

$$\text{Add: } \frac{7}{9x^2} + \frac{x}{3x^2 + 3x} \rightarrow 3x(x+1)$$

$$\frac{\frac{7}{(x+1)}}{\frac{(x+1)}{3x^2}} + \frac{x}{3x(x+1)} = \frac{7}{3x^2} + \frac{3x}{3x(x+1)}$$

LCD:

$$3x^2(x+1)$$

$$\frac{7x+7 + 3x^2}{3x^2(x+1)} = \frac{3x^2 + 7x + 7}{3x^2(x+1)}$$

EXAMPLE 4 Subtract with unlike denominators

$$\text{Subtract: } \frac{x+2}{2x-2} - \frac{-2x-1}{x^2-4x+3}$$

$$\frac{\overbrace{(x-3)}^{(x-3)} \cdot \overbrace{(x+2)}^{(x+2)}}{\overbrace{(x-3)}^{(x-3)} \cdot \underbrace{2(x-1)}_{2(x-1)}} - \frac{\overbrace{(-2x-1)}^{(-2x-1)}}{\underbrace{(x-1)}_{(x-1)} \cdot \underbrace{(x-3)}_{(x-3)}}$$

LCD:

$$2(x-1)(x-3)$$

$$\frac{x^2 - 3x + 2x - 6 + 4x + 2}{2(x-1)(x-3)}$$

$$\frac{x^2 + 3x - 4}{2(x-1)(x-3)} = \frac{(x+4)\cancel{(x-1)}}{2\cancel{(x-1)}(x-3)} = \boxed{\frac{x+4}{2(x-3)}}$$

LIKE DENOMINATORS Perform the indicated operation and simplify.

4. $\frac{x}{16x^2} - \frac{4}{16x^2}$

$$\frac{x-4}{16x^2}$$

FINDING LCMS Find the least common multiple of the polynomials.

10. $2x^2$ and $4x + 12$

$$\underline{2}x^2$$

$$4x+12$$
$$4(x+3)$$
$$\underline{2^2}(x+3)$$

$$2^2 x^2 (x+3)$$

UNLIKE DENOMINATORS Perform the indicated operation and simplify.

$$16. \frac{12}{5x} + \frac{7}{6x} = \frac{6}{6} \cdot \frac{12}{5x} + \frac{5}{5} \cdot \frac{7}{6x}$$

$\frac{2}{2} \cdot \frac{3}{3} \cdot x$

LCD: $5 \cdot 2 \cdot 3x$
 $30x$

$$= \frac{72}{30x} + \frac{35}{30x}$$
$$= \boxed{\frac{107}{30x}}$$