8.5 Add and Subtract Rational Expressions

Simplify $\dfrac{\frac{1}{2} - \frac{2}{5}}{\frac{1}{3} + \frac{3}{4}}$.

Method 1

$\text{top:} \ \dfrac{5}{5}\dfrac{1}{2} - \dfrac{2}{5}\cdot\dfrac{2}{2}$

$\dfrac{5}{10} - \dfrac{4}{10} = \dfrac{1}{10}$

$\text{bottom:} \ \dfrac{4}{4}\dfrac{1}{3} + \dfrac{3}{4}\cdot\dfrac{3}{3}$

$\dfrac{4}{12} + \dfrac{9}{12} = \dfrac{13}{12}$

$\dfrac{\frac{1}{10}}{\frac{13}{12}} = \dfrac{1}{10} \div \dfrac{13}{12} = \dfrac{1}{10} \cdot \dfrac{12}{13}$

$\dfrac{12}{130} = \boxed{\dfrac{6}{65}}$

Method 2

$2, ⑤ ③ 4$

2^2

LCD:

$2^2 \cdot 5 \cdot 3 = 60$

$\dfrac{\frac{1}{2} - \frac{2}{5}}{\frac{1}{3} + \frac{3}{4}} \cdot \dfrac{60}{60}$

$\dfrac{30 - 24}{20 + 45} = \boxed{\dfrac{6}{65}}$

EXAMPLE 6 **Simplify a complex fraction**

Simplify: $\dfrac{\dfrac{5}{x+4}}{\dfrac{1}{x+4}+\dfrac{2}{x}}$

$\dfrac{x}{x}\dfrac{1}{(x+4)}+\dfrac{2}{x}\dfrac{(x+4)}{(x+4)}$

$\dfrac{x+2x+8}{x(x+4)}$

$\dfrac{3x+8}{x(x+4)}$

$\dfrac{\dfrac{5}{x+4}}{\dfrac{3x+8}{x(x+4)}}$

$\dfrac{5}{(x+4)}\cdot\dfrac{x(x+4)}{(3x+8)}$

$\boxed{\dfrac{5x}{3x+8}}$

EXAMPLE 6 **Simplify a complex fraction**

Simplify the complex fraction.

$$\frac{\dfrac{x}{6} - \dfrac{x}{3}}{\dfrac{x}{5} - \dfrac{7}{10}} \cdot \frac{30}{30}$$

$$6 = 2 \cdot 3 \quad 3 \quad 5 \quad 10 = 2 \cdot 5$$

$$LCD:$$
$$2 \cdot 3 \cdot 5 = 30$$

$$\frac{5x - 10x}{6x - 21} = \boxed{\frac{-5x}{3(2x-7)}}$$

EXAMPLE 6 **Simplify a complex fraction**

Simplify the complex fraction.

$$\frac{\frac{2}{x} - 4}{\frac{2}{x} + 3}$$

LCD: X

$$\frac{2-4x}{2+3x} = \boxed{\frac{2(1-2x)}{2+3x}}$$

EXAMPLE 6 **Simplify a complex fraction**

Simplify the complex fraction.

$$\dfrac{\dfrac{3}{x+5}}{\dfrac{2}{x-3}+\dfrac{1}{x+5}}$$

$$\dfrac{(x+5)}{(x+5)}\dfrac{2}{x-3} + \dfrac{1}{x+5}\dfrac{(x-3)}{(x-3)}$$

$$\dfrac{2x+10 + x-3}{(x+5)(x-3)}$$

$$\dfrac{3x+7}{(x+5)(x-3)}$$

$$\dfrac{\dfrac{3}{x+5}}{\dfrac{3x+7}{(x+5)(x-3)}}$$

$$\dfrac{3}{(x+5)} \cdot \dfrac{(x+5)(x-3)}{3x+7}$$

$$\boxed{\dfrac{3(x-3)}{3x+7}}$$

UNLIKE DENOMINATORS Perform the indicated operation(s) and simplify.

28. $\dfrac{x+3}{x^2-2x-8} - \dfrac{x-5}{x^2-12x+32}$

$$\overset{(x-8)}{\underset{\underset{(x-8)}{}}{\dfrac{x+3}{(x-4)(x+2)}}} - \dfrac{x-5}{(x-8)(x-4)}\overset{(x+2)}{\underset{(x+2)}{}}$$

$$\dfrac{(x^2-5x-24) - (x^2-3x-10)}{(x-8)(x-4)(x+2)}$$

$$\dfrac{-2x-14}{(x-8)(x-4)(x+2)} = \boxed{\dfrac{-2(x+7)}{(x-8)(x-4)(x+2)}}$$

UNLIKE DENOMINATORS Perform the indicated operation(s) and simplify.

30. $\dfrac{x+3}{x^2-25} - \dfrac{x-1}{x-5} + \dfrac{3}{x+3}$

$$\dfrac{x+3}{(x-5)(x+5)(x+3)}^{(x+3)} \quad \dfrac{(x+5)\,x-1\,(x+3)}{(x+5)\,x-5\,(x+3)} \quad + \quad \dfrac{3}{x+3}\dfrac{(x+3)(x-5)}{(x+5)(x-5)}$$

$$\dfrac{(x^2+6x+9) - (x^3+7x^2+7x-15) + (3x^2-75)}{(x+5)(x-5)(x+3)}$$

$$\boxed{\dfrac{-x^3-3x^2-x-51}{(x+5)(x-5)(x+3)}} \;\text{ or }\; \boxed{\dfrac{-(x^3+3x+x+51)}{(x+5)(x-5)(x+3)}}$$

SIMPLIFYING COMPLEX FRACTIONS Simplify the complex fraction.

34. $\dfrac{\dfrac{1}{2x-5} - \dfrac{7}{8x-20}}{\dfrac{x}{2x-5}}$ $=$ $\dfrac{\dfrac{-3}{4(2x-5)}}{\dfrac{x}{2x-5}}$ $= \dfrac{-3}{4(2x-5)} \cdot \dfrac{2x-5}{x}$

$$4 \cdot \dfrac{1}{4(2x-5)} - \dfrac{7}{4(2x-5)}$$

$$= \boxed{\dfrac{-3}{4x}}$$

$$\dfrac{4-7}{4(2x-5)} = \dfrac{-3}{4(2x-5)}$$