8.6 Solve Rational Equations

EXAMPLE 1 Solve a rational equation by cross multiplying

Solve: $\frac{3}{x+1} = \frac{9}{4x+5}$

$$\frac{(4x+5)}{(4x+5)}\frac{3}{(x+1)} = \frac{9}{(4x+5)}\frac{(x+1)}{(x+1)}$$

$$3(4x+5) = 9(x+1)$$

$$12x+15 = 9x+9$$

$$-9x \quad -9x$$

$$3x+15 = 9$$

$$-15 \quad -15$$

$$\frac{3x}{3} = \frac{-6}{3}$$

$$x = -2$$

EXAMPLE 1 **Solve a rational equation by cross multiplying**

Solve the equation by cross multiplying. Check your solution(s).

$$\frac{-4}{x+3} = \frac{5}{x-3}$$

$\frac{-4}{-\frac{1}{3}+3}$ ok $= \frac{5}{-\frac{1}{3}-3}$ ok

$-4(x-3) = 5(x+3)$

$-4x+12 = 5x+15$

$+4x \quad +4x$

$12 = 9x+15$

$-15 \quad -15$

$-3 = 9x$

$\frac{-3}{9} = x$

$x = -\frac{1}{3}$

EXAMPLE 1 **Solve a rational equation by cross multiplying**

Solve the equation by cross multiplying. Check your solution(s).

$$\frac{1}{2x+5} = \frac{x}{11x+8}$$

$1(11x+8) = x(2x+5)$

$11x+8 = 2x^2+5x$

$-11x-8 \quad -11x-8$

$0 = 2x^2-6x-8$

$\frac{0}{2} = \frac{2(x^2-3x-4)}{2}$

$0 = x^2-3x-4$

$(x-4)(x+1)$

$x=4$ $x=-1$

EXAMPLE 4 Solve a rational equation with two solutions

What is the solution of $\frac{5}{x} + \frac{7}{4} = -\frac{9}{x}$?

$\frac{4 \cdot 5}{4 \cdot x} + \frac{7 \cdot x}{4 \cdot x} = -\frac{9}{x}$

$\frac{20 + 7x}{4x} = \frac{-9}{x}$

$x(20 + 7x) = -9(4x)$

$20x + 7x^2 = -36x$

$+36x \qquad +36x$

$7x^2 + 56x = 0$

$7x(x + 8) = 0$

$7x = 0 \qquad x + 8 = 0$

$x = 0 \qquad x = -8$

EXAMPLE 4 Solve a rational equation with two solutions

Solve: $1 - \frac{8}{x-5} = \frac{3}{x}$

$\frac{(x-5)1}{(x-5)1} - \frac{8}{x-5} = \frac{3}{x}$

$\frac{x-5-8}{x-5} = \frac{3}{x}$

$\frac{x-13}{x-5} \times \frac{3}{x}$

$x(x-13) = 3(x-5)$

$x^2 - 13x = 3x - 15$

$-3x+15 \quad -3x+15$

$x^2 - 16x + 15 = 0$

$(x-15)(x-1) = 0$

$x = 15$ | $x = 1$

EXAMPLE 5 Check for extraneous solutions

Solve: $\frac{6}{x-3} = \frac{8x^2}{x^2-9} - \frac{4x}{x+3}$ $\neq 0$

$\frac{(x+3)\,6}{(x+3)(x-3)} = \frac{8x^2}{(x-3)(x+3)} - \frac{4x\,(x-3)}{(x+3)(x-3)}$

$\frac{6x+18}{(x+3)(x-3)} = \frac{8x^2-4x^2+12x}{(x+3)(x-3)}$

$6x+18 = 8x^2-4x^2+12x$

$6x+18 = 4x^2+12x$

$-6x \quad -18 \qquad -6x-18$

$0 = 4x^2+6x-18$

$\frac{0}{2} = \frac{2(2x^2+3x-9)}{2}$

$0 = 2x^2+3x-9$

$AC = 2 \cdot -9$

$= -18$

$6 \quad -3$

$0 = 2x^2+6x-3x-9$

$= 2x(x+3)-3(x+3)$

$0 = (2x-3)(x+3)$

$x = \frac{3}{2}$ $x = -3$

Check Solutions!