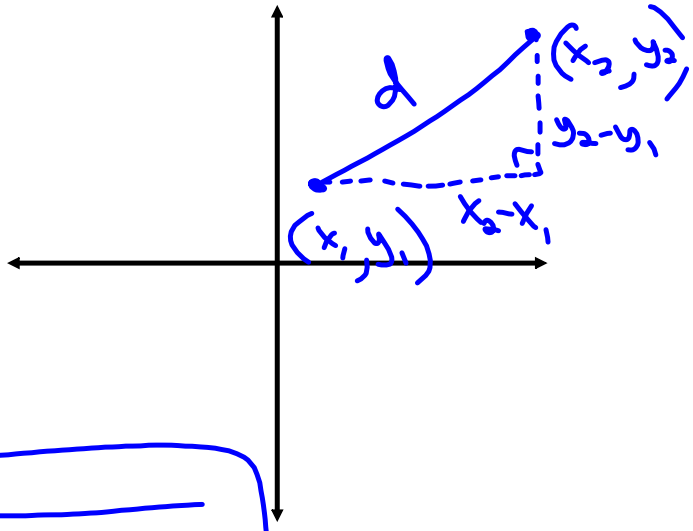
9.1 Distance and Midpoint Formulas and 9.3 Equations of Circles

Distance:

$a^2+b^2=c^2$

$\sqrt{x^2+y^2}=\sqrt{d^2}$

$d=\sqrt{x^2+y^2}$



Distance Formula: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

EXAMPLE 1

(x_1, y_1) (x_2, y_2)

What is the distance between $(-3, 5)$ and $(4, -1)$?

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

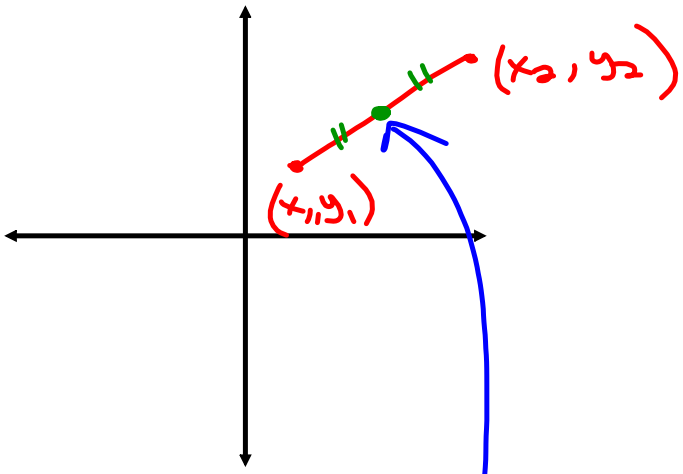
$$d = \sqrt{(4 - (-3))^2 + ((-1) - 5)^2}$$

$$d = \sqrt{(7)^2 + (-6)^2}$$

$$= \sqrt{49 + 36}$$

$$= \boxed{\sqrt{85}}$$

Midpoint:



Midpoint Formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

EXAMPLE 3 Find the midpoint of a line segment

Find the midpoint of the line segment joining (−5, 1) and (−1, 6).

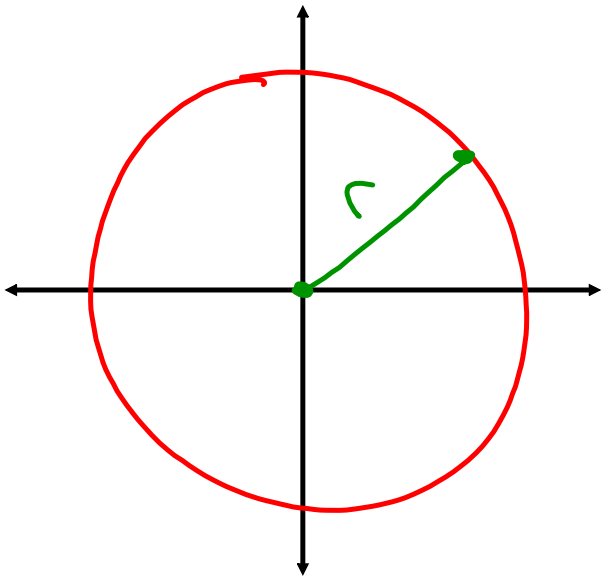
$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M = \left(\frac{-5+-1}{2}, \frac{1+6}{2}\right)$$

$$= \left(\frac{-6}{2}, \frac{7}{2}\right) = \boxed{\left(-3, \frac{7}{2}\right)}$$

Circles: What is a circle? How would you define it?

Equi-Distant



Circles:

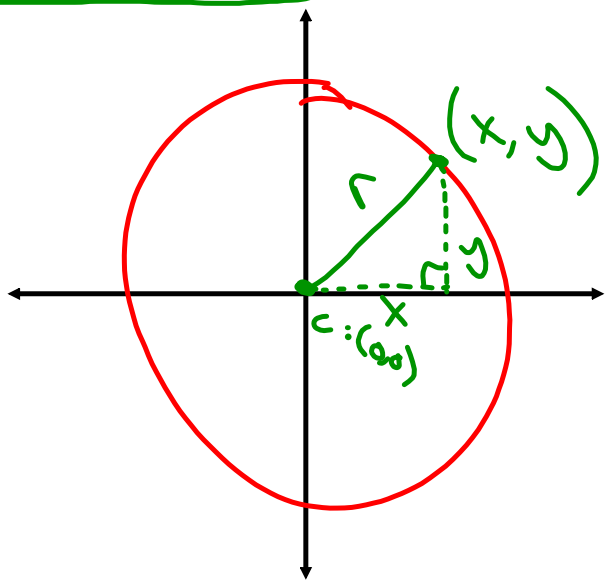
Making Connections

How are the three topics listed below connecting?

1. The Pythagorean Theorem
2. The distance formula
3. The standard equation of a circle

Standard form

$$x^2+y^2=r^2$$



EXAMPLE 1 Graph an equation of a circle

Graph $y^2 = -x^2 + 36$. Identify the radius of the circle.

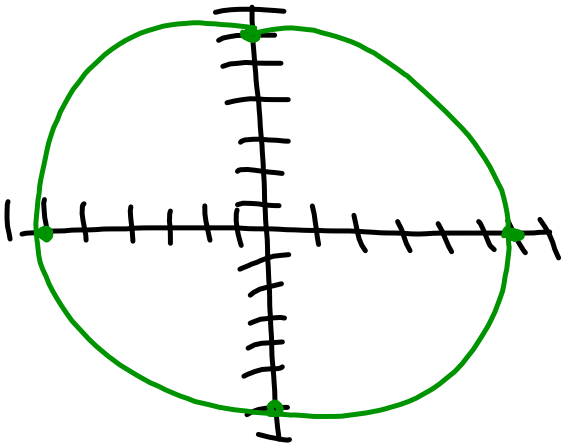
$y^2 = -x^2 + 36$

$+x^2 \quad +x^2$

$x^2 + y^2 = 36$

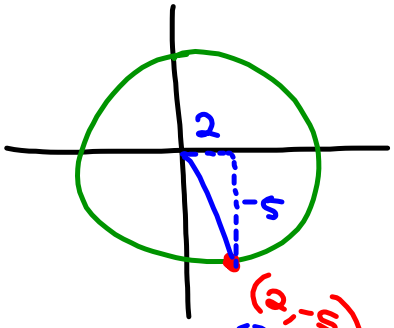
$\sqrt{r^2} = \sqrt{36}$

$r = 6$



EXAMPLE 2 Write an equation of a circle

The point (2, −5) lies on a circle whose center is the origin. Write the standard form of the equation of the circle.



$(2, -5)$

$x^2 + y^2 = r^2$

$(2)^2 + (-5)^2 = r^2$

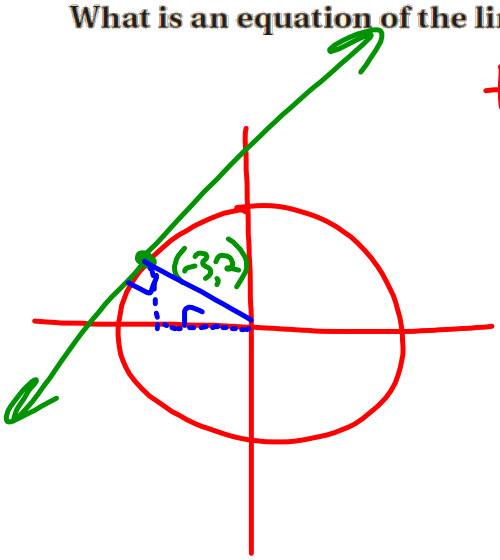
$4 + 25 = r^2$

$29 = r^2$

$x^2 + y^2 = 29$

EXAMPLE 3

What is an equation of the line tangent to the circle $x^2 + y^2 = 13$ at $(-3, 2)$?



touches once

$m_r = \frac{2}{-3}$

$m_\perp = \frac{3}{2}$

$y - y_1 = m(x - x_1)$

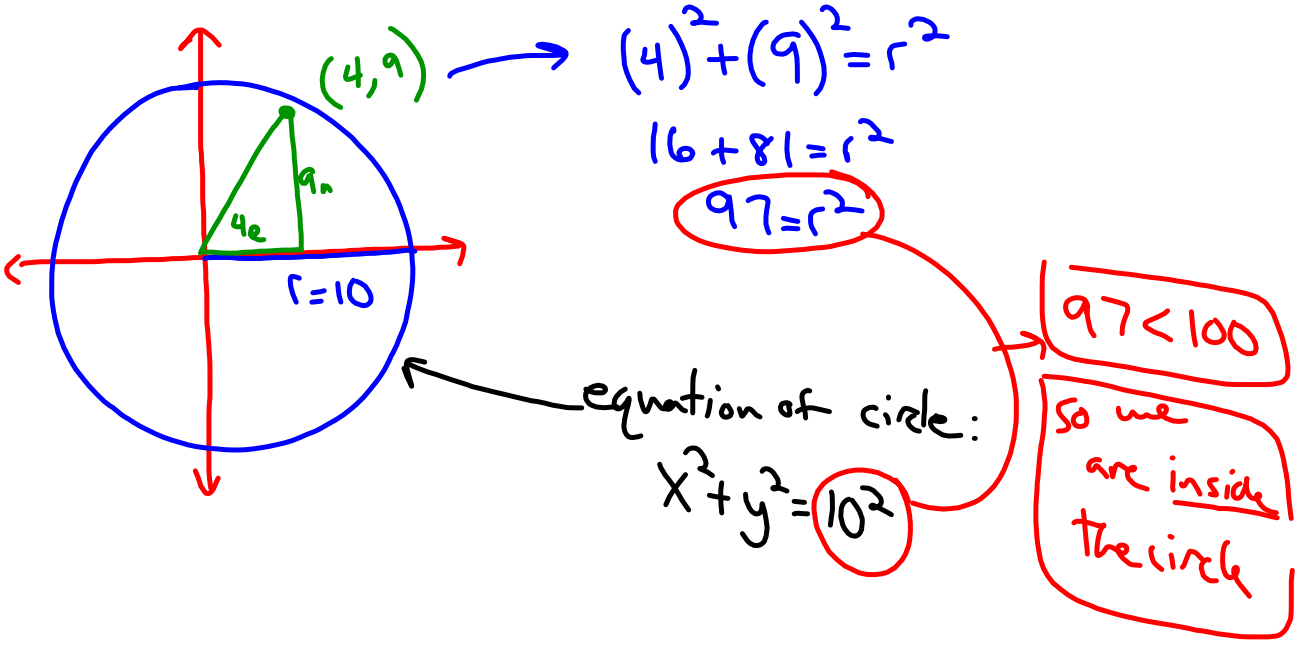
$y - 2 = \frac{3}{2}(x + 3)$

$y - 2 = \frac{3}{2}x + \frac{9}{2} + 2$

$y = \frac{3}{2}x + \frac{13}{2}$

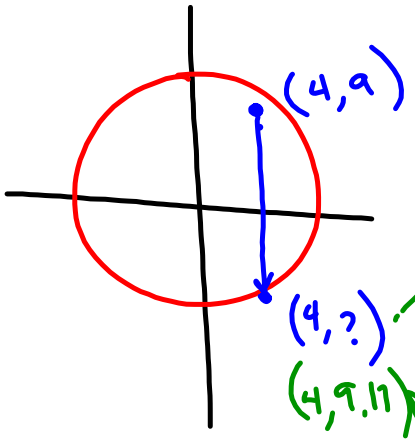
EXAMPLE 4 Write a circular model

CELL PHONES A cellular phone tower services a 10 mile radius. You get a flat tire 4 miles east and 9 miles north of the tower. Are you in the tower's range?



EXAMPLE 5 Apply a circular model

CELL PHONES In Example 4, suppose that you fix your tire and then drive south. For how many more miles will you be in range of the tower?



circle: $x^2+y^2=r^2$

$x^2+y^2=10^2$

$\dashrightarrow x^2+y^2=100$

$4^2+y^2=100$

$16+y^2=100$

$y^2=84$

$\therefore y=\sqrt{84}=2\sqrt{21}\approx 9.17$

$d = 9+9.17 = \boxed{18.17}$