AP Calc AB

Derivative Review

Name: Key

Hour:

Find the first derivative of each of the following.

1) $y = \frac{2-x}{3x+1}$

$\frac{dy}{dx} = \boxed{\frac{(-1)(3x+1)-(2-x)(3)}{(3x+1)^2}}$

2) $y = \frac{2}{(5x+1)^3} = 2(5x+1)^{-3}$

$\frac{dy}{dx} = \boxed{-6(5x+1)^{-4}(5)}$

3) $f(x) = \frac{x}{\sqrt{1-x^2}}$

$f'(x) = \boxed{\frac{(1)(\sqrt{1-x^2})-(x)(\frac{1}{2}(1-x^2)^{-1/2}(-2x))}{1-x^2}}$

4) $f(x) = (x^2-2)(x^{-1}+2)$

$f'(x) = \boxed{(2x)(x^{-1}+2)+(x^2-2)(-1x^{-2})}$

5) $y = 3x^2 + \frac{2}{x} - \frac{5}{x^2}$

$\frac{dy}{dx} = \boxed{6x - 2x^{-2} + 10x^{-3}}$

6) $g(x) = x^8 - 3\sqrt{x} + 5x^{-3}$

$g'(x) = \boxed{8x^7 - \frac{3}{2}x^{-\frac{1}{2}} - 15x^{-4}}$

7) $f(x) = (x-1)^2\sqrt{3x+1}$

$f'(x) = \boxed{2(x-1)\sqrt{3x+1} + (x-1)^2(\frac{1}{2}(3x+1)^{-\frac{1}{2}})(3)}$

8) $h(x) = \left(\frac{2x-4}{x^2}\right)^3$

$h'(x) = \boxed{3\left(\frac{2x-4}{x^2}\right)^2\left(\frac{(2)(x^2)-(2x-4)(2x)}{x^4}\right)}$

9) $y = (1+\sec x)(x^2 - \tan x)$

$\frac{dy}{dx} = \boxed{(\sec x\tan x)(x^2-\tan x)+(1+\sec x)(2x-\sec^2 x)}$

10) $f(x) = \cot\left(\frac{\csc(2x)}{x^3+5}\right)$

$f'(x) = \boxed{-\csc^2\left(\frac{\csc(2x)}{x^3+5}\right)\left(\frac{-\csc(2x)\cot(2x)(2)(x^3+5)-\csc(2x)(3x^2)}{(x^3+5)^2}\right)}$

11) $y = \frac{1}{2x+\sin^3 x} = (2x+\sin^3 x)^{-1}$

$\frac{dy}{dx} = \boxed{-1(2x+\sin^3 x)^{-2}(2+3(\sin x)^2(\cos x))}$

12) $k(x) = \ln(2x)$

$\boxed{k'(x) = \frac{1}{2x}(2)}$

13) $y = \log(\sqrt[3]{x+1})$

$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt[3]{x+1}\,\ln(10)}\left(\frac{1}{3}(x+1)^{-2/3}\right)}$

14) $f(x) = 2xe^{\sqrt{x}}$

$\boxed{f'(x) = (2)(e^{\sqrt{x}})+(2x)(e^{\sqrt{x}})(\frac{1}{2}x^{-\frac{1}{2}})}$

15) $y = 2^{\sin^{-1} x}$

$\boxed{\frac{dy}{dx} = 2^{\sin^{-1}x}(\ln(2))\left(\frac{1}{\sqrt{1-x^2}}\right)}$

16) $g(x) = \sec^{-1}(2x+1)$

$\boxed{g'(x) = \frac{1}{|2x+1|\sqrt{(2x+1)^2-1}}(2)}$

17) $y = \sqrt{\tan^{-1}(x^2)}$

$\boxed{\frac{dy}{dx} = \frac{1}{2}(\tan^{-1}(x^2))^{-\frac{1}{2}}\left(\frac{1}{(x^2)^2+1}\right)(2x)}$

18) $f(x) = \log_2(\cos^{-1}(4^x))$

$f'(x) = \boxed{\frac{1}{\cos^{-1}(4^x)\ln(2)}\left(\frac{-1}{\sqrt{1-(4^x)^2}}\right)(4^x\ln(4))}$

Use logarithmic differentiation to find $\frac{dy}{dx}$ for each of the following.

19) $y = \sqrt[3]{\frac{x^3-1}{x^3+1}}$

$\ln(y) = \frac{1}{3}\ln\left(\frac{x^3-1}{x^3+1}\right)$

$= \frac{1}{3}\left(\ln(x^3-1) - \ln(x^3+1)\right)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{3}\left(\frac{1}{x^3-1}(3x^2) - \frac{1}{x^3+1}(3x^2)\right)$

$\boxed{\frac{dy}{dx} = \left(\sqrt[3]{\frac{x^3-1}{x^3+1}}\right)\left(\frac{1}{3}\left(\frac{3x^2}{x^3-1} - \frac{3x^2}{x^3+1}\right)\right)}$

20) $y = x^{\csc x}$

$\ln(y) = \ln(x^{\csc x})$

$\ln(y) = \csc x \ln(x)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = (-\csc(x)\cot(x))\ln(x) + (\csc x)\left(\frac{1}{x}\right)$

$\boxed{\frac{dy}{dx} = (x^{\csc x})\left(-\csc(x)\cot(x)\ln(x) + \frac{\csc x}{x}\right)}$

21) Find the equation of the normal line for $y = (x^2+1)(x+1)^3$ when $x = -1$.

$\frac{dy}{dx} = (2x)(x+1)^3 + (x^2+1)(3(x+1)^2)$

$\frac{dy}{dx} = -2(0) + \cancel{(2)(0)}$ $m=0$

normal: $\frac{1}{0}$ = undef.

$\boxed{x=-1}$

22) At what x-value is $y = 3x - 1$ tangent to $f(x) = x^3 + 1$?

$f'(x) = 3x^2$

$3x^2 = 3$

$x^2 = 1$ $x = \pm 1$

$3(1)-1=2$

$3(-1)-1=-4$

$(-1)^3+1=0$

$(1)^3+1=2$

$\boxed{x=1}$

For #23-26, use the table below to find the indicated value.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	−1	3	$\frac{3}{2}$
4	2	−1	5	1
6	2	1	4	−2

23) Given $h(x) = f(x) - g(x)$, find $h'(2)$.

$h'(x) = f'(x) - g'(x)$

$(-1) - \left(\frac{3}{2}\right)$

$\boxed{h'(2) = -\frac{5}{2}}$

24) Given $p(x) = f(x) \cdot g(x)$, find $p'(4)$.

$p'(x) = f'(x)g(x) + f(x)g'(x)$

$(-1)(5) + (2)(1)$

$\boxed{p'(4) = -3}$

25) Given $q(x) = f(x) \div g(x)$, find $q'(6)$.

$q'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$

$q'(6) = \frac{(1)(4) - (2)(-2)}{(4)^2} = \frac{8}{16} = \frac{1}{2}$

$\boxed{q'(6) = \frac{1}{2}}$

26) Given $c(x) = g^2(f(x))$, find $c'(4)$.

$c'(x) = 2(g(f(x)))(g'(f(x)))(f'(x))$

$= 2(g(2))(g'(2))(-1)$

$2(3)\left(\frac{3}{2}\right)(-1)$

$\boxed{c'(4) = -9}$