1) $\boxed{f(x) = \dfrac{1+x^2}{1-x^2}}$

Dom: $(-\infty,-1) \cup (-1,1) \cup (1,+\infty)$

Inc: $(0,1) \cup (1,+\infty)$

Dec: $(-\infty,-1) \cup (-1,0)$

min: $(0,1)$

max: none

c. up: $(-1,1)$

c. down: $(-\infty,-1) \cup (1,+\infty)$

Inf. pts: none

$\lim_{x \to \infty} f(x)$: -1

$\lim_{x \to -\infty} f(x)$: -1

v-asymp.: $x = \pm 1$

y-int: $(0,1)$

$$f'(x) = \frac{(2x)(1-x^2) - (1+x^2)(-2x)}{(1-x^2)}$$

$$= \boxed{\frac{4x}{(1-x^2)^2}}$$

$x=0$

$x \neq -1, 1$

$f'(x)$: $-$ -1 $-$ 0 $+$ 1 $+$

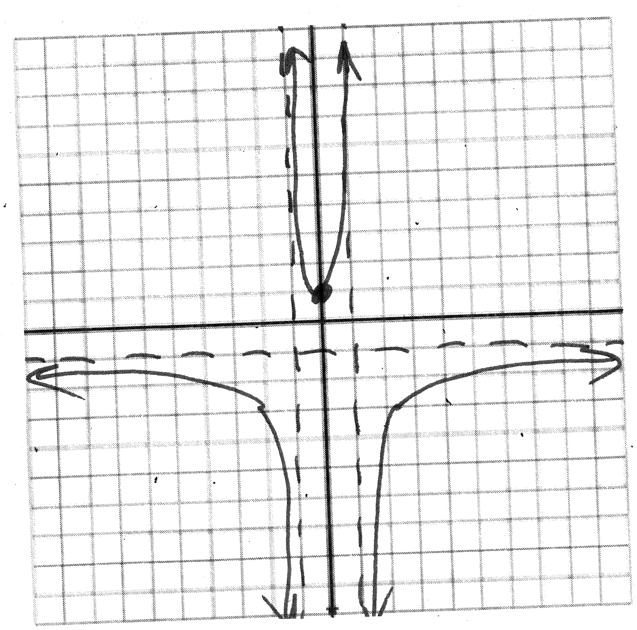
$$f''(x) = \frac{(4)(1-x^2)^2 - (4x)(2(1-x^2))(-2x)}{(1-x^2)^4}$$

$$= \frac{4(1-x^2)((1-x^2)+4x^2)}{(1-x^2)^4}$$

$$= \boxed{\frac{4(1+3x^2)}{(1-x^2)^3}}$$

$x \neq \pm 1$

$f''(x)$: $-$ -1 $+$ 1 $-$



2) $\boxed{f(x) = \dfrac{x}{(x-1)^2}}$

Dom: $(-\infty, 1) \cup (1, +\infty)$

Inc: $(-1, 1)$

Dec: $(-\infty, -1) \cup (1, +\infty)$

min: $(-1, -\frac{1}{4})$

max: none

c. up: $(-2, 1) \cup (1, +\infty)$

c. down: $(-\infty, -2)$

Inf. pts: $(-2, \frac{-2}{9})$

$\lim\limits_{x \to \infty} f(x)$: 0

$\lim\limits_{x \to -\infty} f(x)$: 0

V-asymp: $x = 1$

y-int: $(0, 0)$

$$f'(x) = \frac{(1)(x-1)^2 - (x)(2(x-1))}{(x-1)^4}$$

$$= \frac{(x-1)((x-1) - 2x)}{(x-1)^4}$$

$$= \boxed{\frac{-(x+1)}{(x-1)^3}}$$

$x = -1$

$x \neq 1$

$f'(x)$: $-$ | -1 | $+$ | 1 | $-$

$$f''(x) = \frac{(-1)(x-1)^3 + (x+1)(3(x-1)^2)}{(x-1)^6}$$

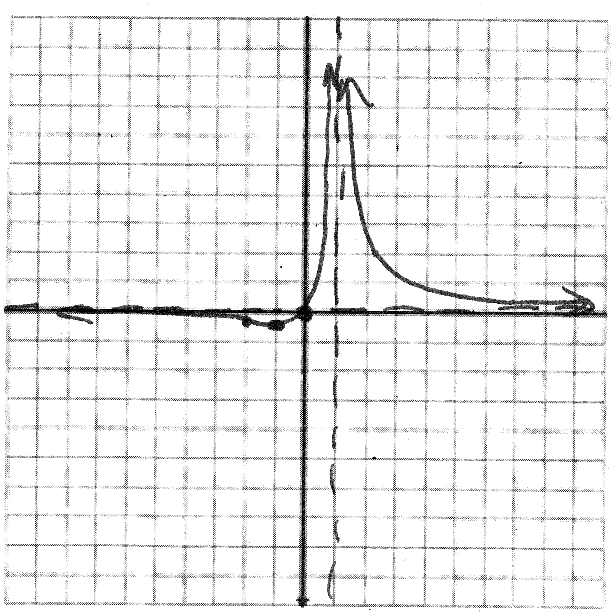
$$= \frac{(x-1)^2(-(x-1) + 3(x+1))}{(x-1)^6}$$

$$= \boxed{\frac{2(x+2)}{(x-1)^4}}$$

$x = -2$

$x \neq 1$

$f''(x)$: $-$ | -2 | $+$ | 1 | $+$



3) $\boxed{f(x) = \sqrt{x^2+1} - x}$

Dom: $(-\infty, +\infty)$

Inc: none

Dec: $(-\infty, +\infty)$

min: none

max: none

c. up: $(-\infty, +\infty)$

c. down: none

Inf. pts: none

$\lim_{x\to\infty} f(x)$: 0

$\lim_{x\to-\infty} f(x)$: $+\infty$

y-int: $(0,1)$

$x(x^2+1)^{-\frac{1}{2}}$

$$f'(x) = \frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x) - 1$$

$$= \frac{x}{\sqrt{x^2+1}} - \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}$$

$$= \frac{x - \sqrt{x^2+1}}{\sqrt{x^2+1}} \quad \begin{matrix} =0 \\ \neq 0 \end{matrix}$$

$x = \sqrt{x^2+1}$

$x^2 = x^2+1$

$f'(x)$ — 0

$$f''(x) = (1)(x^2+1)^{-\frac{1}{2}} + (x)\left(-\frac{1}{2}(x^2+1)^{-\frac{3}{2}}(2x)\right)$$

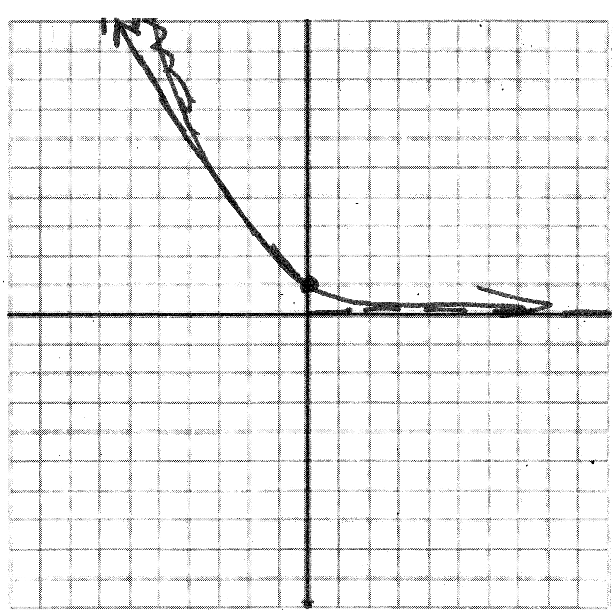
$$= \frac{1}{\sqrt{x^2+1}} - \frac{x^2}{\sqrt{x^2+1}^3}$$

$$= \frac{\cancel{x^2}+1-\cancel{x^2}}{\sqrt{x^2+1}^3} = \boxed{\frac{1}{\sqrt{x^2+1}^3}}$$

$f''(x)$ + 0

$$\lim_{x\to+\infty} \sqrt{x^2+1} - x = \lim_{x\to+\infty} \frac{\frac{\sqrt{x^2+1}}{x} - 1}{\frac{1}{x}} \overset{H}{=} \lim_{x\to+\infty} \frac{\frac{\left(\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)\right)(x) - (\sqrt{x^2+1})(1)}{\cancel{x^2}}}{-\frac{1}{\cancel{x^2}}}$$

$$= \lim_{x\to\infty}\left(\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}\right) = \lim_{x\to\infty} \frac{1}{\sqrt{x^2+1}} = 0$$



4) $\boxed{f(x) = x\tan x}$

$-\frac{\pi}{2} < x < \frac{\pi}{2}$

Dom: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Inc: $\left(0, \frac{\pi}{2}\right)$

Dec: $\left(-\frac{\pi}{2}, 0\right)$

min: $(0,0)$

max: none

c. up: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

c. down: none

Inf. pts.: none

$\lim_{x\to\frac{\pi}{2}^-} f(x)$: $+\infty$

$\lim_{x\to-\frac{\pi}{2}^+} f(x)$: $+\infty$

v-asymp: $x = \pm\frac{\pi}{2}$

y-int: $(0,0)$

$f'(x) = (1)(\tan x) + (x)(\sec^2 x)$

$\frac{\sin x}{\cos x} + \frac{x}{\cos^2 x}$

$\boxed{\frac{\cos x \sin x + x}{\cos^2 x}} = 0$

$x = 0$

$x \neq \frac{\pi}{2}, -\frac{\pi}{2}$

$f'(x)$: $-\frac{\pi}{2}$ ... $-$ ... 0 ... $+$ ... $\frac{\pi}{2}$

$f''(x) = \sec^2 x + (1)(\sec^2 x) + (x)(2\sec x)(\sec x \tan x)$

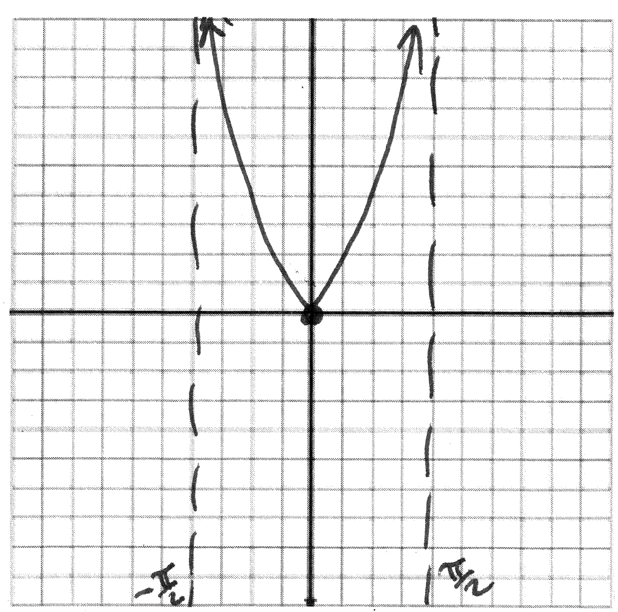
$= 2\sec^2 x + 2x\sec^2 x \tan x$

$= \boxed{2\sec^2 x\,(1 + x\tan x)} = 0$

$x\tan x = -1$

none

$f''(x)$: $-\frac{\pi}{2}$ ... $+$ ... 0 ... $\frac{\pi}{2}$



5) $\boxed{f(x) = e^{\frac{-1}{x+1}}}$

Dom: $(-\infty, -1) \cup (-1, +\infty)$

Inc: $(-\infty, -1) \cup (-1, +\infty)$

Dec: none

min: none

max: none

c. up: $(-\infty, -1) \cup (-1, -\frac{1}{2})$

c. down: $(-\frac{1}{2}, +\infty)$

Inf. pts: $(-\frac{1}{2}, \frac{1}{e^2})$

$\lim_{x \to \infty} f(x)$: 1

$\lim_{x \to -\infty} f(x)$: 1

y-int: $(0, \frac{1}{e})$

$\lim_{x \to -1^+} f(x)$: 0

$\lim_{x \to -1^-} f(x)$: ∞

$$f'(x) = \boxed{e^{\frac{-1}{x+1}}\left(\frac{1}{(x+1)^2}\right)} = \frac{1}{e^{\frac{1}{x+1}}(x+1)^2}$$

$x \neq -1$

$f'(x)$: $+$ | -1 | $+$

$$f''(x) = \frac{\left(e^{\frac{-1}{x+1}}\right)\left((x+1)^{-2}\right)(x+1)^2 - \left(e^{\frac{-1}{x+1}}\right)\left(2(x+1)\right)}{(x+1)^4}$$

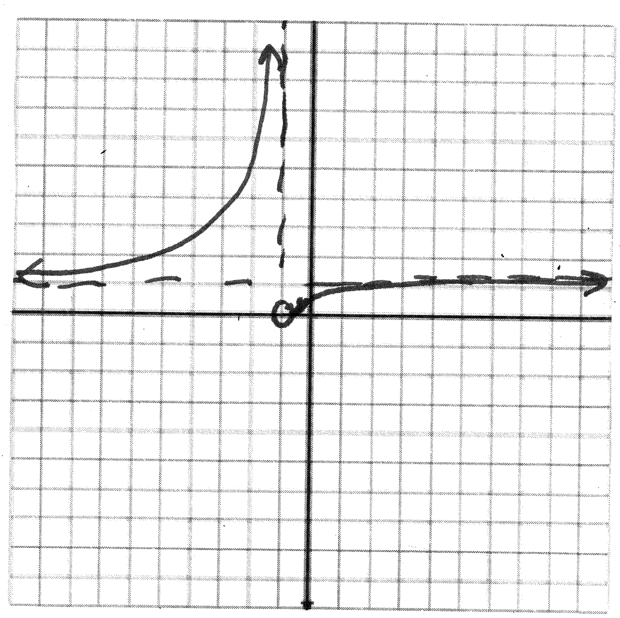
$$= \frac{\left(e^{\frac{-1}{x+1}}\right)\left(1 - 2(x+1)\right)}{(x+1)^4}$$

$$= \boxed{\frac{\left(e^{\frac{-1}{x+1}}\right)(-2x-1)}{(x+1)^4}} = 0$$

$x = -\frac{1}{2}$

$x \neq -1$

$f''(x)$: $+$ | -1 | $-\frac{1}{2}$ | $-$



6) $\boxed{f(x) = \ln(\tan^2(x))}$

Dom: $\mathbb{R}$, $x \neq \frac{\pi}{2}k$ where k is any int

Inc: $\ldots \cup (-\pi, -\frac{\pi}{2}) \cup (0, \frac{\pi}{2}) \cup (\pi, \frac{3\pi}{2}) \cup \ldots$

Dec: $\ldots \cup (-\frac{3\pi}{2}, -\pi) \cup (-\frac{\pi}{2}, 0) \cup (\frac{\pi}{2}, \pi) \cup \ldots$

min: none

max: none

c. up: $\ldots \cup (-\frac{3\pi}{4}, -\frac{\pi}{2}) \cup (-\frac{\pi}{2}, -\frac{\pi}{4}) \cup (\frac{\pi}{4}, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \frac{3\pi}{4}) \cup \ldots$

c. down: $\ldots \cup (-\frac{\pi}{4}, 0) \cup (0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, \pi) \cup (\pi, \frac{5\pi}{4}) \cup \ldots$

Inf. Pts: $(\frac{\pi}{4} + \frac{\pi}{2}k, 0)$ where k is any int.

y-int: none

$$f'(x) = \frac{1}{\tan^2(x)}(2\tan(x))(\sec^2(x))$$

$$= \frac{2\sec^2(x)}{\tan(x)} = \frac{\frac{2}{\cos^2 x}}{\frac{\sin x}{\cos x}}$$

$$= \boxed{\frac{2}{\sin x \cos x}} \quad x \neq \frac{\pi}{2}k$$

$-$ $+$ $-$ $+$ $-$ $+$ $-$

$-\frac{3\pi}{2}$ $-\pi$ $-\frac{\pi}{2}$ 0 $\frac{\pi}{2}$ π $\frac{3\pi}{2}$

$$f''(x) = -2(\sin x \cos x)^{-2}\left((\cos x)(\cos x) + (\sin x)(-\sin x)\right)$$

$$= \boxed{\frac{-2(\cos^2 x - \sin^2 x)}{(\sin x \cos x)^2}} = 0$$

$$x = \frac{\pi}{4} + \frac{\pi}{2}k$$

$x \neq \frac{\pi}{2}k$

$f''(x)$: $+$ $-$ $+$ $+$ $-$ $-$ $+$ $+$ $-$ $-$

$-\pi$ $-\frac{3\pi}{4}$ $-\frac{\pi}{2}$ $-\frac{\pi}{4}$ 0 $\frac{\pi}{4}$ $\frac{\pi}{2}$ $\frac{3\pi}{4}$ π