AP Calc AB | Integration Review | Name: key | Hour: _____

For each integral below, decide which of the following types it is: nothing needed (basic antiderivative), algebra or trig. Identity needed (u-think), substitution (u-sub), integration by parts (u-cry), or it can't be done with the techniques we've covered. Then evaluate each integral (except for the last type of course).

1. $\int (x^3+1)\,dx$ basic

$\frac{1}{4}x^4+x+C$

2. $\int_0^1 x^2(x^3+1)^4\,dx$ u-sub

$u=x^3+1$
$du=3x^2dx$
$\frac{1}{3}\int_1^2 u^4\,du = \frac{1}{3}\left(\frac{1}{5}u^5\right)\Big|_1^2$
$\frac{1}{15}(32-1) = \frac{31}{15}$

3. $\int \sqrt{x^3+1}\,dx$

N/A

4. $\int (x^3+1)^2\,dx$ u-think

$\int (x^6+2x^3+1)\,dx$
$\frac{1}{7}x^7+\frac{1}{2}x^4+x+C$

5. $\int_4^9 \sqrt{x}(1-x^2)\,dx$ u-think

$\int_4^9 \left(x^{\frac{1}{2}}-x^{\frac{5}{2}}\right)dx$
$\left(\frac{2}{3}x^{\frac{3}{2}}-\frac{2}{7}x^{\frac{7}{2}}\right)\Big|_4^9 = \left(\frac{2}{3}(27)-\frac{2}{7}(2187)\right)-\left(\frac{2}{3}(8)-\frac{2}{7}(128)\right)$
$\left(\frac{126}{7}-\frac{4374}{7}\right)-\left(\frac{112}{21}-\frac{768}{21}\right)$
$-\frac{12744}{21}+\frac{656}{21}$
$-\frac{12088}{21}$

6. $\int \sqrt{1-x^2}\,dx$

N/A

7. $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}}$ basic

$\arcsin(x)\Big|_{-\frac{1}{2}}^{\frac{1}{2}}$
$\arcsin\left(\frac{1}{2}\right)-\arcsin\left(-\frac{1}{2}\right) = \frac{\pi}{6}-\left(-\frac{\pi}{6}\right) = \frac{\pi}{3}$

8. $\int \frac{x\,dx}{\sqrt{1-x^2}}$ u-sub

$u=1-x^2 \quad du=-2x\,dx$
$-\frac{1}{2}\int \frac{du}{\sqrt{u}} = \frac{1}{2}\left(2u^{\frac{1}{2}}\right)+C$
$-\sqrt{1-x^2}+C$

Answers:
1) $\frac{1}{4}x^4+x+C$ 2) $\frac{31}{15}$ 3) N/A 4) $\frac{1}{7}x^7+\frac{1}{2}x^4+x+C$ 5) $-\frac{12088}{21}$ 6) N/A 7) $\frac{\pi}{3}$ 8) $-\sqrt{1-x^2}+C$

9. $\int \cos^2(x)\sin^3(x)\,dx$

N/A

10. $\int_0^{\pi} \sqrt{1-\cos^2(x)}\,dx$ u-think

$\int_0^{\pi} \sqrt{\sin^2(x)}\,dx = \int_0^{\pi} \sin(x)\,dx = -\cos(x)\Big|_0^{\pi}$

$-(-1)-(-1) = 1+1 = 2$

11. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{\cos^2(x)}$ u-think

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2(x) = \tan(x)\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = 1-(-1)$

2

12. $\int x^2\cos(x)\,dx$ parts

$u = x^2$	$v = \sin(x)$
$u = 2x\,dx$	$v = \cos(x)\,dx$

$x^2\sin(x) - 2\int x\sin(x)\,dx$

$u = x$	$v = -\cos(x)$
$du = dx$	$dv = \sin(x)\,dx$

$x^2\sin(x) - 2\left(-x\cos(x) + \int \cos(x)\,dx\right)$

$x^2\sin(x) + 2x\cos(x) - 2\sin(x) + C$

13. $\int \tan(x)\sec(x)\,dx$ basic

$\sec(x) + C$

14. $\int \tan(x)\cos(x)\,dx$ u-think

$\int \frac{\sin(x)}{\cos(x)}\cos(x)\,dx = \int \sin(x)\,dx$

$-\cos(x) + C$

15. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sec^2(x)}{\sqrt{\tan(x)}}\,dx$ u-sub

$u = \tan(x)$

$du = \sec^2(x)\,dx$

$\int_{\frac{1}{\sqrt{3}}}^{1} \frac{du}{\sqrt{u}} = (2u^{\frac{1}{2}})\Big|_{\frac{1}{\sqrt{3}}}^{1} = 2\left(1-\sqrt{\tfrac{1}{\sqrt{3}}}\right)$

$2 - \frac{2}{\sqrt{3}}$

16. $\int \frac{dx}{\tan(x)+1}$

N/A

17. $\int 3xe^x\,dx$ parts

$u = x$	$v = e^x$
$du = dx$	$dv = e^x dx$

$3\left(xe^x - \int e^x dx\right)$

$3xe^x - 3e^x + C$

18. $\int_0^{\ln(5)} \frac{e^x}{3+e^x}\,dx$ u-sub

$u = 3+e^x$

$du = e^x dx$

$\int_4^8 \frac{du}{u} = \ln|u|\Big|_4^8 = \ln(8) - \ln(4)$

$\ln(2)$

19. $\int (e^x + 3)\,dx$ basic

$e^x + 3x + C$

20. $\int \frac{\ln(e^{2x})}{x^2}\,dx$ u-think

$\int \frac{2x}{x^2}\,dx = \int \frac{2}{x}\,dx = 2\ln|x| + C$

Answers:

9) N/A 10) 2 11) 2 12) $x^2\sin(x) + 2x\cos(x) - 2\sin(x) + C$ 13) $\sec(x) + C$ 14) $-\cos(x) + C$
15) $2 - \frac{2}{\sqrt{3}}$ 16) N/A 17) $3xe^x - 3e^x + C$ 18) $\ln(2)$ 19) $e^x + 3x + C$ 20) $2\ln|x| + C$