Lawrence High School

Math Department

Summer Review

For Students Entering

Calculus AB/BC

Please show your work on the following problems. This packet will be collected by your math teacher so don't forget to bring your finished work back with you in August. If you need assistance on a topic check out: https://www.wolframalpha.com/, https://www.khanacademy.org/, http://www.purplemath.com/, or google the topic.

Fractional & Negative Exponents

Simplify using only positive exponents.

1. $-3x^{-3}$

2. $-5\left(\frac{3}{2}\right)(4-9x)^{-\frac{1}{2}}(-9)$

3. $2\left(\frac{2}{2-x}\right)\left(\frac{-2}{(2-x)^2}\right)$

4. $(16x^2y)^{\frac{3}{4}}$

5. $-\frac{x^{-\frac{1}{2}}}{2}\sin\sqrt{x}$

6. $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

7. $-4\left(\frac{2x-1}{2x+1}\right)^{-3}\left(\frac{2(2x+1)-2(2x-1)}{(2x+1)^2}\right)$

8. $\frac{\frac{1}{2}(2x+5)^{-\frac{3}{2}}}{\frac{3}{2}}$

9. $\left(\frac{1}{x^{-2}}+\frac{4}{x^{-1}y^{-1}}+\frac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

Domain

Find the domain of the following functions.

10. $y = \frac{3x-2}{4x+1}$

11. $y = \frac{x^2-4}{2x+4}$

12. $y = \frac{x^2-5x-6}{x^2-3x-18}$

13. $y = \frac{2^{2-x}}{x}$

14. $y = \sqrt{x-3} - \sqrt{x+3}$

15. $y = \frac{\sqrt{2x-9}}{2x+9}$

16. $y = \frac{x^2+8x+12}{\sqrt[4]{x+5}}$

17. $y = \sqrt{x^2-5x-14}$

18. $y = \frac{\sqrt[3]{x-6}}{\sqrt{x^2-x-30}}$

19. $y = \log(2x-12)$

20. $y = \sqrt{\tan x}$

21. $y = \frac{x}{\cos x}$

Absolute Value Equations and Inequalities

Write the following absolute value equations as piecewise equations.

22. $y = |2x - 4|$

23. $y = |6 - 2x| + 1$

24. $y = |4x + 1| + 2x - 3$

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Solve the following absolute value inequalities.

25. $|x - 3| > 12$

26. $|x - 3| \leq 4$

27. $|10x + 8| > 2$

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28. $|3x - 4| > 2$

29. $|x - 6| > x + 8$

30. $|x + 1| \leq |x - 3|$

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Quadratic Absolute Value Equations and Inequalities

Solve the following. (You may want to use a number line.)

31. $|x^2 - 1| = 0$

32. $|x^2 + x - 12| = 2$

33. $|x^2 + 4x + 4| > 5$

34. $x^2 - 16 > 0$

35. $x^2 + 6x - 16 > 0$

36. $|x^2 - 3x| \geq 10$

37. $2x^2 + 4x \leq 3$

38. $x^3 + 4x^2 - x \geq 4$

39. $2\sin^2 x \geq \sin x \quad 0 \leq x < 2\pi$

Special Factorizations

Factor completely.

40. $x^3 + 8$

41. $x^3 - 64$

42. $27x^3 - 125y^3$

43. $x^4 + 11x^2 - 80$

44. $ac + cd - ab - bd$

45. $2x^2 + 50y^2 - 20xy$

46. $x^2 + 12x + 36 - 9y^2$

47. $x^3 - xy^2 + x^2y - y^3$

48. $(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$

Function Transformations

Describe in words what the following would do to the graph of $f(x)$.

49. $g(x) = f(x) - 4$
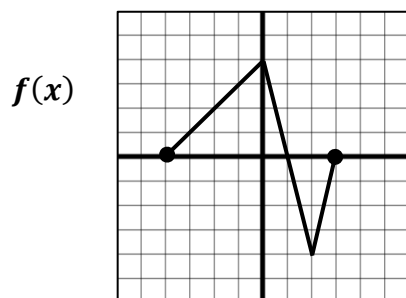
50. $h(x) = f(x - 4)$

51. $k(x) = -f(x + 2)$

52. $m(x) = 5f(x) + 3$

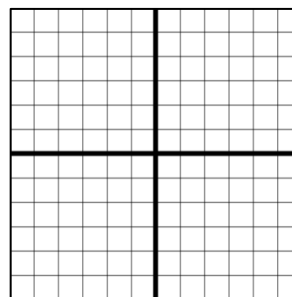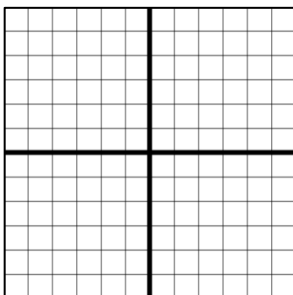53. $j(x) = \frac{1}{2}f(3x)$

54. $p(x) = |f(x)|$

Use the graph of the function $f(x)$ below to graph the other functions.
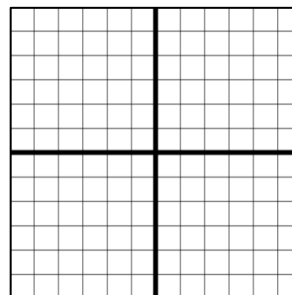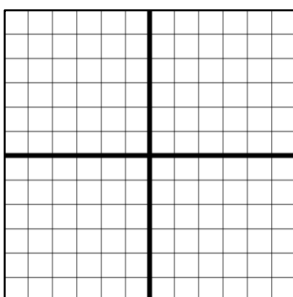
$f(x)$

55. $g(x) = f(x) + 3$
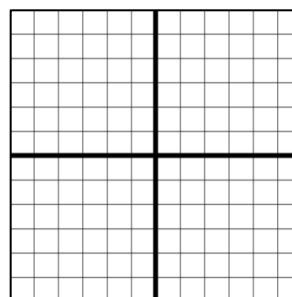
56. $h(x) = -f(x - 2)$

57. $k(x) = f(x + 1) - 4$

58. $m(x) = |f(x)|$

59. $n(x) = f(|x|)$

Even and Odd Functions

Determine algebraically if the relation is an even or odd function, or neither.

60. $f(x) = 2x^2 - 7$

61. $f(x) = -4x^3 - 2x$

62. $f(x) = 4x^2 - 4x + 4$

63. $f(x) = x - \frac{1}{x}$

64. $f(|x|) = |x| - x^2 + 1$

65. $5x^2 - 6y = 1$

66. $y = e^x - \frac{1}{e^x}$

67. $3y^3 = 4x^3 + 1$

68. $3x = |y|$

Solving Quadratic Equations by Various Methods

Solve each equation by any algebraic method.

69. $7x^2 - 3x = 0$

70. $4x(x-2) - 5x(x-1) = 2$

71. $x^2 + 6x + 4 = 0$

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72. $2x^2 - 3x + 3 = 0$

73. $2x^2 - (x+2)(x-3) = 12$

74. $x + \frac{1}{x} = \frac{13}{6}$

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75. $x^4 - 9x^2 + 8 = 0$

76. $x - 10\sqrt{x} + 9 = 0$

77. $\frac{1}{x^2} - \frac{1}{x} = 6$

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Asymptotes and Intercepts

Find the vertical and horizontal asymptotes and the x-intercepts or state they don't exist.

78. $y = \dfrac{x}{x-3}$

79. $y = \dfrac{x+4}{x^2-1}$

80. $y = \dfrac{x-4}{x^2+1}$

81. $y = \dfrac{x^2-2x+1}{x^2-3x-4}$

82. $y = \dfrac{x^2-9}{x^3+3x^2-18x}$

83. $y = \dfrac{2x^2+6x}{x^3-3x^2-4x}$

84. $y = \dfrac{x^2-x-6}{x^3-6x^2+x-6}$

85. $y = \dfrac{2x^3}{x^3-1}$

86. $y = \dfrac{\sqrt{x}}{2x^2-10}$

Complex Fractions

Simplify the following.

87. $\dfrac{x}{x-\dfrac{1}{2}}$

88. $\dfrac{\dfrac{1}{x}+4}{\dfrac{1}{x}-2}$

89. $\dfrac{x-\dfrac{1}{x}}{x+\dfrac{1}{x}}$

90. $\dfrac{\dfrac{3}{x}-\dfrac{4}{y}}{\dfrac{4}{x}-\dfrac{3}{y}}$

91. $\dfrac{1-\dfrac{2}{3x}}{x-\dfrac{4}{9x}}$

92. $\dfrac{\dfrac{x^2-y^2}{xy}}{\dfrac{x+y}{y}}$

93. $\dfrac{x^{-3}-x}{x^{-2}-1}$

94. $\dfrac{\dfrac{x}{1-x}+\dfrac{1+x}{x}}{\dfrac{1-x}{x}+\dfrac{x}{1+x}}$

95. $\dfrac{\dfrac{4}{x-5}+\dfrac{2}{x+2}}{\dfrac{2x}{x^2-3x-10}+3}$

Composition of Functions

If $f(x) = x^2$, $g(x) = 2x - 1$, and $h(x) = 2^x$, find the following

96. $f(g(2))$

97. $f(g(x))$

98. $f(h(-1))$

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99. $h(f(-1))$

100. $g\left(f\left(h\left(\frac{1}{2}\right)\right)\right)$

101. $g(f(x))$

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102. $g(f(2))$

103. $g(g(x))$

104. $f(h(x))$

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Solving Ration Equations

Solve each equation by any algebraic method.

105. $\dfrac{2}{3} - \dfrac{5}{6} = \dfrac{1}{x}$

106. $x + \dfrac{6}{x} = 5$

107. $\dfrac{x+1}{3} - \dfrac{x-1}{2} = 1$

108. $\dfrac{x-5}{x+1} = \dfrac{3}{5}$

109. $\dfrac{60}{x} - \dfrac{60}{x-5} = \dfrac{2}{x}$

110. $\dfrac{2}{x+5} + \dfrac{1}{x-5} = \dfrac{16}{x^2-25}$

111. $\dfrac{x}{x-2} + \dfrac{2x}{4-x^2} = \dfrac{5}{x+2}$

112. $\dfrac{x}{2x-6} - \dfrac{3}{x^2-6x+9} = \dfrac{x-2}{3x-9}$

113. $\dfrac{2x+3}{x-1} = \dfrac{10}{x^2-1} + \dfrac{2x-3}{x+1}$

Solving Trigonometric Equations

Solve each equation on the interval $[0, 2\pi)$

114. $\sin x = \dfrac{1}{2}$

115. $\cos^2 x = \cos x$

116. $2\cos x + \sqrt{3} = 0$

117. $4\sin^2 x = 1$

118. $2\sin^2 x + \sin x = 1$

119. $\cos^2 x + 2\cos x = 3$

120. $2\sin x \cos x + \sin x = 0$

121. $8\cos^2 x - 2\cos x = 1$

122. $\sin x - \cos x = 0$

The Difference Quotient

Recall from Precalculus that the difference quotient is defined as:

$$\frac{f(x+h) - f(x)}{h}; \quad h \neq 0$$

Evaluate the difference quotient of the following functions, simplifying as much as possible.

123. $f(x) = 2x + 5$

124. $f(x) = x^2 - 3$

125. $f(x) = 4x^2 + 3x - 7$

126. $f(x) = x^3 - 8x$

127. $f(x) = \frac{1}{x+3}$

128. $f(x) = \sqrt{x} + 5$

129. $f(x) = \sqrt{2x - 1}$

130. $f(x) = \sin x$

131. $f(x) = e^x$