

2.7 Exploring Transformations

MATERIALS • graphing calculator

QUESTION How are the equation and the graph of an absolute value function related?

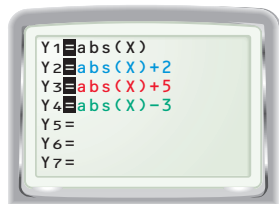
You can investigate families of *absolute value functions* with equations of the form $y = a|x - h| + k$ by varying the values of a , h , and k and then graphing. The resulting graphs are *transformations* of the graph of the parent function $y = |x|$.

EXAMPLE 1 Graph $y = |x| + k$

Graph and describe the family of absolute value functions of the form $y = |x| + k$.

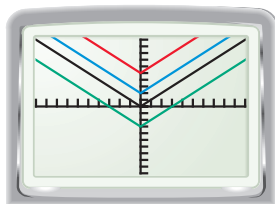
STEP 1 *Vary the value of k*

Enter $y = |x|$, $y = |x| + 2$,
 $y = |x| + 5$, and $y = |x| - 3$.



STEP 2 *Display graphs*

Graph the equations in the standard viewing window by pressing **ZOOM** **6**.



STEP 3 *Compare graphs*

Describe how the family of graphs of $y = |x| + k$ is related to the graph of $y = |x|$.

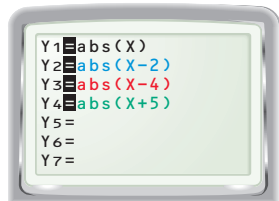
The graphs of absolute value functions of the form $y = |x| + k$ have the same shape as the graph of $y = |x|$, but are shifted k units vertically.

EXAMPLE 2 Graph $y = |x - h|$

Graph and describe the family of absolute value functions of the form $y = |x - h|$.

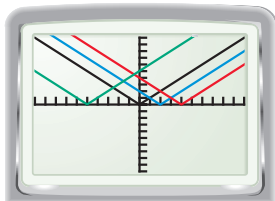
STEP 1 *Vary the value of h*

Enter $y = |x|$, $y = |x - 2|$,
 $y = |x - 4|$, and $y = |x + 5|$.



STEP 2 *Display graphs*

Graph the equations in the standard viewing window by pressing **ZOOM** **6**.



STEP 3 *Compare graphs*

Describe how the family of graphs of $y = |x - h|$ is related to the graph of $y = |x|$.

The graphs of absolute value functions of the form $y = |x - h|$ have the same shape as the graph of $y = |x|$, but are shifted h units horizontally.

1 PLAN AND PREPARE

Explore the Concept

- Students will explore transformations of the graph of $y = |x|$.
- This activity leads into the study of deriving new absolute value functions from the parent graph in Examples 1–3 in Lesson 2.7.

Materials

Each student will need a graphing calculator.

Recommended Time

Work activity: 10 min
Discuss results: 5 min

Grouping

Students can work individually or in pairs. If students work in pairs, they can take turns graphing the equations.

2 TEACH

Tips for Success

Make sure students know how to find the “abs” notation on their calculators and remember to enclose x in parentheses when entering an equation of the form $y = |x| + k$ in their graphing calculator.

Key Questions

- When is the graph of $y = |x| + k$ shifted upward from the graph of $y = |x|$? **when $k > 0$**
- When is the graph of $y = |x - h|$ shifted to the left of the graph of $y = |x|$? **when $h < 0$**

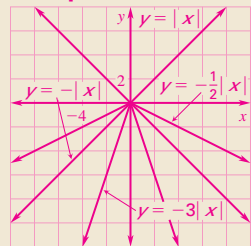
Key Discovery

The graphs of $y = |x| + k$, $y = |x - h|$ and $y = a|x - h| + k$ are transformations of the graph of $y = |x|$.

3 ASSESS AND RETEACH

1. What is the value of a when the graph of $y = a|x - h| + k$ has the same shape as the graph of $y = |x|$? **1**
2. What is the value of a when the graph of $y = a|x|$ is a reflection in the x -axis of the graph of $y = |x|$? **-1**

1. Step 2:



Step 3: When $a < 0$, $y = a|x|$ has its highest point at the origin. If $-1 < a < 0$, the graph is wider than $y = |x|$. If $a < -1$, the graph is narrower than $y = |x|$.

2–10. See Additional Answers beginning on p. AA1.

13. If $a < 0$, then the graph is reflected over the x -axis. If $-1 < a < 1$, then the graph is wider than $y = |x|$. If $a < -1$ or $a > 1$, then the graph is narrower than $y = |x|$.

14. (h, k) ; if $a < 0$ it is the highest point and if $a > 0$ it is the lowest point.

EXAMPLE 3 Graph $y = a|x|$ where a is a positive number

Graph and describe the family of absolute value functions of the form $y = a|x|$ where $a > 0$.

STEP 1 Vary the value of a

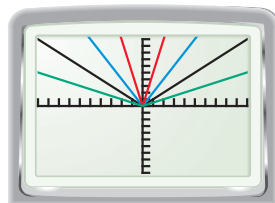
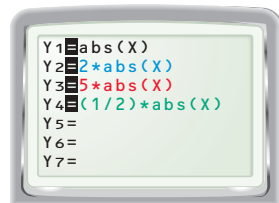
Enter $y = |x|$, $y = 2|x|$, $y = 5|x|$, and $y = \frac{1}{2}|x|$.

STEP 2 Display graphs

Graph the equations in the standard viewing window by pressing **ZOOM** **6**.

STEP 3 Compare graphs

Describe how the family of graphs of $y = a|x|$ where $a > 0$ is related to the graph of $y = |x|$.



As with $y = |x|$, the graph of $y = a|x|$ ($a > 0$) has its lowest point at the origin. If $a > 1$, the graph is narrower than that of $y = |x|$. If $0 < a < 1$, the graph is wider than that of $y = |x|$.

PRACTICE

1. Graph and describe the family of absolute value functions of the form $y = a|x|$ where $a < 0$. Follow these steps:

STEP 1 Enter $y = |x|$, $y = -|x|$, $y = -3|x|$, and $y = -\frac{1}{2}|x|$. **Check students' work.**

STEP 2 Graph the equations in the standard viewing window by pressing **ZOOM** **6**. **See margin.**

STEP 3 Describe how the family of graphs of $y = a|x|$ where $a < 0$ is related to the graph of $y = |x|$. **See margin.**

Describe how the graph of the given equation is related to the graph of $y = |x|$.

Then graph the given equation along with $y = |x|$ to confirm your answer. 2–10. See margin for art.

2. $y = |x| + 6$
translated up 6 units

3. $y = |x| - 4$
translated down 4 units

4. $y = |x - 3|$
translated right 3 units

5. $y = |x + 2|$
translated left 2 units

6. $y = \frac{2}{3}|x|$
wider than $y = |x|$

7. $y = -6|x|$
reflected over x -axis and narrower than $y = |x|$

8. $y = |x - 1| + 2$
translated up 2 units and right 1 unit

9. $y = 3|x + 2|$
narrower than $y = |x|$ and translated left 2 units

10. $y = -0.5|x + 1| + 7$
reflected over the x -axis and wider than $y = |x|$, translated left 1 unit and up 7 units

DRAW CONCLUSIONS

Answer the following questions about the graph of $y = a|x - h| + k$.

11. How does the value of k affect the graph? **shifts the graph vertically**
12. How does the value of h affect the graph? **shifts the graph horizontally**
13. How do the sign and absolute value of a affect the graph? **See margin.**
14. What are the coordinates of the lowest or highest point on the graph? How can you tell whether this point is the lowest point or the highest point? **See margin.**