

Extra Example 6.1

Evaluate the expression.

a. $(\sqrt[5]{-32})^2$ **4**

b. $64^{5/3}$ **1024**

REVIEW KEY VOCABULARY

- n th root of a , p. 414
- index of a radical, p. 414
- simplest form of a radical, p. 422
- like radicals, p. 422
- power function, p. 428
- composition, p. 430
- inverse relation, p. 438
- inverse function, p. 438
- radical function, p. 446
- radical equation, p. 452

VOCABULARY EXERCISES

- Copy and complete: The index of the radical $\sqrt[4]{7}$ is 4 .
- List two different pairs of like radicals. **Sample answer:** $2\sqrt[3]{x}$ and $-3\sqrt[3]{x}$, $\sqrt{xy+1}$ and $3\sqrt{xy+1}$
- Copy and complete: A(n) power function has the form $y = ax^b$ where a is a real number and b is a rational number. **power**
- WRITING** Explain how the graph of a function and the graph of its inverse are related. **They are reflections of each other in the line $y = x$.**
- WRITING** Explain how to use the horizontal line test to determine whether the inverse of a function f is also a function. **If a horizontal line crosses the graph of the function more than once, the inverse is not a function.**
- WRITING** Describe how the graph of $y = \sqrt[3]{x} - 4 + 5$ is related to the graph of the parent function $y = \sqrt[3]{x}$. **It is translated 4 units right and 5 units up.**
- REASONING** A student began solving the equation $x^{2/3} = 5$ by cubing each side. What will the student have to do next? What could the student have done to solve the equation in just one step? **Take the square root of each side; raise each side to the $\frac{3}{2}$ power.**

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 6.

6.1 Evaluate n th Roots and Use Rational Exponents

pp. 414–419

EXAMPLE

Evaluate the expression.

a. $(\sqrt[4]{16})^5 = 2^5 = 32$

b. $27^{-4/3} = \frac{1}{27^{4/3}} = \frac{1}{(27^{1/3})^4} = \frac{1}{3^4} = \frac{1}{81}$

EXERCISES

Evaluate the expression without using a calculator.

8. $81^{1/4}$ **3**

9. $0^{1/3}$ **0**

10. $\sqrt[3]{-64}$ **-4**

11. $\sqrt[3]{125}$ **5**

12. $256^{3/4}$ **64**

13. $27^{-2/3}$ **$\frac{1}{9}$**

14. $(\sqrt[3]{8})^7$ **128**

15. $\frac{1}{(\sqrt[5]{-32})^{-3}}$ **-8**

EXAMPLE 2
on p. 415
for Exs. 8–15

6.2 Apply Properties of Rational Exponents

pp. 420–427

EXAMPLE

Write the expression in simplest form. Assume all variables are positive.

$$\text{a. } \sqrt[3]{48} = \sqrt[3]{8 \cdot 6} = \sqrt[3]{8} \cdot \sqrt[3]{6} = 2\sqrt[3]{6} \qquad \text{b. } \left(\frac{x^4}{y^8}\right)^{1/2} = \frac{(x^4)^{1/2}}{(y^8)^{1/2}} = \frac{x^{4 \cdot 1/2}}{y^{8 \cdot 1/2}} = \frac{x^2}{y^4}$$

EXERCISES

Write the expression in simplest form. Assume all variables are positive.

$$16. \sqrt[3]{80} \quad 2\sqrt[3]{10} \qquad 17. (3^4 \cdot 5^4)^{-1/4} \quad \frac{1}{15} \qquad 18. (25a^{10}b^{16})^{1/2} \quad 5a^5b^8 \qquad 19. \sqrt{\frac{18x^5y^4}{49xz^3}} \quad \frac{3x^2y^2\sqrt{2z}}{7z^2}$$

EXAMPLES 4, 6, and 7

on pp. 422–423
for Exs. 16–19

6.3 Perform Function Operations and Composition

pp. 428–434

EXAMPLE

Let $f(x) = 3x^2 + 1$ and $g(x) = x + 4$. Perform the indicated operation.

$$\begin{aligned} \text{a. } f(x) + g(x) &= (3x^2 + 1) + (x + 4) = 3x^2 + x + 5 \\ \text{b. } f(x) \cdot g(x) &= (3x^2 + 1)(x + 4) = 3x^3 + 12x^2 + x + 4 \\ \text{c. } f(g(x)) &= f(x + 4) = 3(x + 4)^2 + 1 = 3(x^2 + 8x + 16) + 1 = 3x^2 + 24x + 49 \end{aligned}$$

EXERCISES

Let $f(x) = 4x - 6$ and $g(x) = x + 8$. Perform the indicated operation.

$$20. f(x) + g(x) \quad 5x + 2 \qquad 21. f(x) - g(x) \quad 3x - 14 \qquad 22. f(x) \cdot g(x) \quad 4x^2 + 26x - 48 \qquad 23. f(g(x)) \quad 4x + 26$$

EXAMPLES 1, 2, and 5

on pp. 428–430
for Exs. 20–23

6.4 Use Inverse Functions

pp. 438–445

EXAMPLE

Find the inverse of the function $y = 3x + 7$.

$$y = 3x + 7 \quad \text{Write original function.}$$

$$x = 3y + 7 \quad \text{Switch } x \text{ and } y.$$

$$x - 7 = 3y \quad \text{Subtract 7 from each side.}$$

$$\frac{1}{3}x - \frac{7}{3} = y \quad \text{Divide each side by 3.}$$

EXERCISES

Find the inverse of the function.

$$24. y = \frac{1}{3}x + 4 \quad y = \frac{x-4}{2} \qquad 25. y = 4x^2 + 9, x \geq 0 \quad y = \frac{\sqrt{x-9}}{2} \qquad 26. f(x) = x^3 - 4 \quad f^{-1}(x) = \sqrt[3]{x+4}$$

EXAMPLES 1, 4, and 5

on pp. 438–441
for Exs. 24–26

Extra Example 6.2

Write the expression in simplest form. Assume all variables are positive.

$$\text{a. } \sqrt[3]{250} \quad 5\sqrt[3]{2} \qquad \text{b. } \sqrt[5]{\frac{c}{d^8}} \quad \frac{\sqrt[5]{cd^8}}{d^2}$$

Extra Example 6.3

Let $f(x) = x^2 - 1$ and $g(x) = 2x + 5$. Perform the indicated operation.

$$\begin{aligned} \text{a. } f(x) - g(x) & \quad x^2 - 2x - 6 \\ \text{b. } f(x) \cdot g(x) & \quad 2x^3 + 5x^2 - 2x - 5 \\ \text{c. } g(f(x)) & \quad 2x^2 + 3 \end{aligned}$$

Extra Example 6.4

Find the inverse of the function

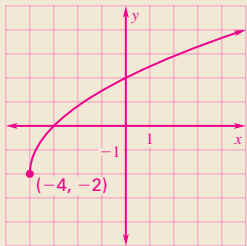
$$f(x) = 2x^3 + 5. \quad f^{-1}(x) = \sqrt[3]{\frac{x-5}{2}}$$

6

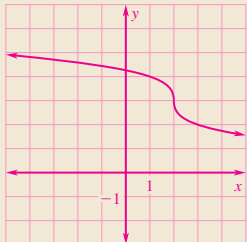
CHAPTER REVIEW

Extra Examples 6.5

1. Graph
- $y = 2\sqrt{x+4} - 2$
- .

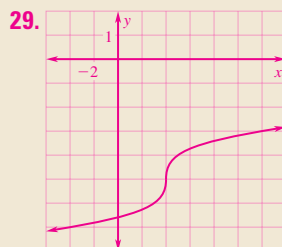
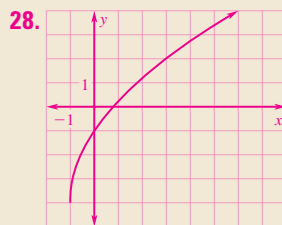
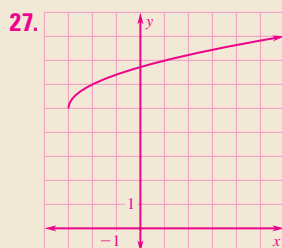


2. Graph
- $y = -\sqrt[3]{x-2} + 3$
- .



Extra Example 6.6

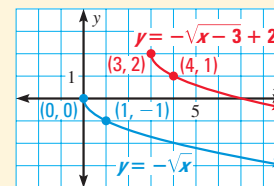
- Solve
- $\sqrt[3]{5x-14} = -4$
- .
- 10**



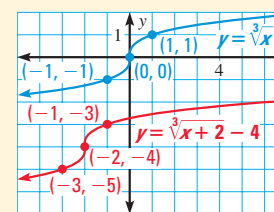
6.5 Graph Square Root and Cube Root Functions

pp. 446–451

EXAMPLE

Graph $y = -\sqrt{x-3} + 2$.Sketch the graph of $y = -\sqrt{x}$. Notice that it begins at the origin and passes through the point (1, -1).For $y = -\sqrt{x-3} + 2$, $h = 3$, and $k = 2$. So, shift the graph of $y = -\sqrt{x}$ right 3 units and up 2 units. The resulting graph begins at the point (3, 2) and passes through the point (4, 1).

EXAMPLE

Graph $y = \sqrt[3]{x+2} - 4$.Sketch the graph of $y = \sqrt[3]{x}$. Notice that it passes through the points (-1, -1), (0, 0), and (1, 1).For $y = \sqrt[3]{x+2} - 4$, $h = -2$ and $k = -4$. So, shift the graph of $y = \sqrt[3]{x}$ left 2 units and down 4 units. The resulting graph passes through the points (-3, -5), (-2, -4), and (-1, -3).

EXAMPLES 4 and 5

on p. 448
for Exs. 27–29

EXERCISES

Graph the function. Then state the domain and range. 27–29. See margin for art.

27. $y = \sqrt{x+3} + 5$

domain: $x \geq -3$, range: $y \geq 5$

28. $y = 3\sqrt{x+1} - 4$

domain: $x \geq -1$, range: $y \geq -4$

29. $y = \sqrt[3]{x-4} - 5$

domain: all real numbers,
range: all real numbers

6.6 Solve Radical Equations

pp. 452–459

EXAMPLE

Solve $\sqrt{4x+9} = 5$.

$$\sqrt{4x+9} = 5 \quad \text{Write original equation.}$$

$$(\sqrt{4x+9})^2 = 5^2 \quad \text{Square each side to eliminate the radical.}$$

$$4x+9 = 25 \quad \text{Simplify.}$$

$$4x = 16 \quad \text{Subtract 9 from each side.}$$

$$x = 4 \quad \text{Divide each side by 4.}$$

CHECK Check $x = 4$ in the original equation.

$$\sqrt{4x+9} = \sqrt{4(4)+9} = \sqrt{25} = 5 \quad \checkmark$$

EXAMPLES 1, 3, and 5

on pp. 452–454
for Exs. 30–32

EXERCISES

Solve the equation. Check for extraneous solutions.

30. $\sqrt[3]{5x-4} = 2$ **$2\frac{2}{5}$**

31. $3x^{3/4} = 24$ **16**

32. $\sqrt{x^2-10} = \sqrt{3x}$ **-2, 5**