

Convert each rectangular equation to polar form.

1) $x^2 = 12$

$$(r \cos \theta)^2 = 12$$

$$r^2 \cos^2 \theta = 12$$

$$r^2 = \frac{12}{\cos^2 \theta}$$

$$r^2 = 12 \sec^2 \theta \text{ or } r = \sqrt{12} \sec \theta$$

2) $x^2 + y^2 = 7$

$$r^2 = 7$$

$$r = \sqrt{7}$$

3) $3x - 2y = 1$

$$3(r \cos \theta) - 2(r \sin \theta) = 1$$

$$r(3 \cos \theta - 2 \sin \theta) = 1$$

$$r = \frac{1}{3 \cos \theta - 2 \sin \theta}$$

4) $x - 3y = 1$

$$r(\cos \theta) - 3r(\sin \theta) = 1$$

$$r(\cos \theta - 3 \sin \theta) = 1$$

$$r = \frac{1}{\cos \theta - 3 \sin \theta}$$

5) $2x^2 + 2y^2 + 5x = 0$

$$2(x^2 + y^2) + 5x = 0$$

$$2r^2 + 5r \cos \theta = 0$$

$$2r^2 = -5r \cos \theta$$

$$r^2 = -\frac{5}{2} r \cos \theta$$

$$r = -\frac{5}{2} \cos \theta$$

6) $3x^2 + 3y^2 - 2x = 0$

$$3(x^2 + y^2) - 2x = 0$$

$$3r^2 - 2r \cos \theta = 0$$

$$3r^2 = 2r \cos \theta$$

$$r = \frac{2}{3} \cos \theta$$

7) $x^2 + (y - 2)^2 - 4 = 0$

$$x^2 + y^2 - 4y + 4 - 4 = 0$$

$$x^2 + y^2 - 4y = 0$$

$$r^2 - 4r \sin \theta = 0$$

$$r^2 = 4r \sin \theta$$

$$r = 4 \sin \theta$$

8) $(x - 5)^2 + y^2 - 25 = 0$

$$x^2 - 10x + 25 + y^2 - 25 = 0$$

$$x^2 + y^2 - 10x = 0$$

$$r^2 - 10r \cos \theta = 0$$

$$r = 10 \cos \theta$$

9) $x = y$

$$r \cos \theta = r \sin \theta$$

$$\cos \theta = \sin \theta$$

$$1 = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\theta = \frac{\pi}{4}$$

10) $x = 7$

$$r \cos \theta = 7$$

$$r = \frac{7}{\cos \theta}$$

$$r = 7 \sec \theta$$

11) $x^2 + y^2 = 4$

$$r^2 = 4$$

$$r = 2$$

12) $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$$36 \left(\frac{(r \cos \theta)^2}{9} + \frac{(r \sin \theta)^2}{4} = 1 \right)$$

$$4r^2 \cos^2 \theta + 9r^2 \sin^2 \theta = 36$$

$$r^2(4 \cos^2 \theta + 9 \sin^2 \theta) = \frac{36}{4 \cos^2 \theta + 9 \sin^2 \theta}$$

13) $y^2 = 4x$

$$r^2 \sin^2 \theta = 4r \cos \theta$$

$$r = \frac{4 \cos \theta}{\sin^2 \theta}$$

$$r = 4 \frac{1}{\sin \theta} \cdot \frac{\cos \theta}{\sin \theta}$$

$$r = 4 \csc \theta \cot \theta$$

14) $x^2 + xy + y^2 = 1$

$$x^2 + y^2 + xy = 1$$

$$r^2 + (r \cos \theta)(r \sin \theta) = 1$$

$$r^2 + r^2 \cos \theta \sin \theta = 1$$

$$r^2(1 + \cos \theta \sin \theta) = 1$$

$$r^2 = \frac{1}{1 + \cos \theta \sin \theta}$$

15) $(x - 3)^2 + (y + 1)^2 = 4$

$$x^2 - 6x + 9 + y^2 + 2y + 1 = 4$$

$$x^2 + y^2 - 6x + 2y + 6 = 0$$

$$r^2 - 6r \cos \theta + 2r \sin \theta + 6 = 0$$

$$r^2 + (-6 \cos \theta + 2 \sin \theta)r + 6 = 0$$

$$r = 3 \cos \theta - \sin \theta \pm \sqrt{4 \cos^2 \theta - 3 \sin^2 \theta - 1}$$

Convert each polar equation to rectangular form.

16) $r = 7 \tan \theta \sec \theta$

$$r = 7 \left(\frac{\sin \theta}{\cos \theta} \right) \left(\frac{1}{\cos \theta} \right)$$

$$r \cos^2 \theta = 7 \sin \theta$$

$$r^2 \cos^2 \theta = 7r \sin \theta$$

$$\boxed{x^2 = 7y}$$

19) $r = \frac{1}{4 - \cos \theta}$

$$4r - r \cos \theta = 1$$

$$\boxed{4\sqrt{x^2 + y^2} - x = 1}$$

22) $r \cos \theta + r \sin \theta = 1$

$$\boxed{x + y = 1}$$

25) $r^2 \sin(2\theta) = 2$

$$2r^2 \sin \theta \cos \theta = 2$$

$$(r \sin \theta)(r \cos \theta) = 1$$

$$y \cdot x = 1$$

$$\boxed{y = \frac{1}{x}}$$

28) $r = 8 \sin \theta$

$$r^2 = 8r \sin \theta$$

$$\boxed{x^2 + y^2 = 8y}$$

17) $r = \csc \theta \cot \theta$

$$r = \left(\frac{1}{\sin \theta} \right) \left(\frac{\cos \theta}{\sin \theta} \right)$$

$$r \sin^2 \theta = \cos \theta$$

$$r^2 \sin^2 \theta = r \cos \theta$$

$$\boxed{y^2 = x}$$

20) $r = \sin \theta$

$$r^2 = r \sin \theta$$

$$\boxed{x^2 + y^2 = y}$$

23) $r^2 = 1$

$$\boxed{x^2 + y^2 = 1}$$

26) $r = \csc \theta e^{r \cos \theta}$

$$r \sin \theta = e^{r \cos \theta}$$

$$\boxed{y = e^x}$$

29) $r = 2 \cos \theta + 2 \sin \theta$

$$r^2 = 2r \cos \theta + 2r \sin \theta$$

$$\boxed{x^2 + y^2 = 2x + 2y}$$

18) $r = \frac{1}{3 + \sin \theta}$

$$3r + r \sin \theta = 1$$

$$\boxed{3\sqrt{x^2 + y^2} + y = 1}$$

21) $r = 4 \csc \theta$

$$r = \frac{4}{\sin \theta}$$

$$r \sin \theta = 4$$

$$\boxed{y = 4}$$

24) $r^2 = 4r \sin \theta$

$$\boxed{x^2 + y^2 = 4y}$$

27) $r \sin \theta = \ln(r) + \ln(\cos \theta)$

$$r \sin \theta = \ln(r \cos \theta)$$

$$\boxed{y = \ln(x)}$$

30) $r \sin\left(\theta + \frac{\pi}{6}\right) = 2$

$$r \left(\sin(\theta) \cos\left(\frac{\pi}{6}\right) + \cos(\theta) \sin\left(\frac{\pi}{6}\right) \right) = 2$$

$$r \sin(\theta) \left(\frac{\sqrt{3}}{2}\right) + r \cos(\theta) \left(\frac{1}{2}\right) = 2$$

$$\boxed{\frac{\sqrt{3}}{2} y + \frac{1}{2} x = 2}$$