Separable Differential Equations

Find the general solution of each differential equation.

1) $\dfrac{dy}{dx} = e^{x-y}$

$e^y = e^x + C$

$y = \ln\left(e^x + C\right)$

2) $\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y}$

$\tan y = x + C$

$y = \tan^{-1}(x + C)$

3) $\dfrac{dy}{dx} = xe^y$

$-e^{-y} = \dfrac{x^2}{2} + C_1$

$y = -\ln\left(-\dfrac{x^2}{2} + C\right)$

4) $\dfrac{dy}{dx} = \dfrac{2x}{e^{2y}}$

$\dfrac{e^{2y}}{2} = x^2 + C_1$

$y = \dfrac{\ln\left(2x^2 + C\right)}{2}$

5) $\dfrac{dy}{dx} = 2y - 1$

$\dfrac{\ln|2y-1|}{2} = x + C_1$
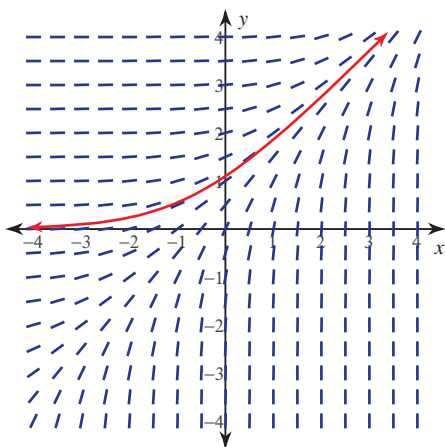
$y = \dfrac{Ce^{2x} + 1}{2}$

6) $\dfrac{dy}{dx} = 2yx + yx^2$

$\ln|y| = x^2 + \dfrac{x^3}{3} + C_1$

$y = Ce^{x^2 + \frac{x^3}{3}}$

For each problem, find the particular solution of the differential equation that satisfies the initial condition. You may use a graphing calculator to sketch the solution on the provided graph.
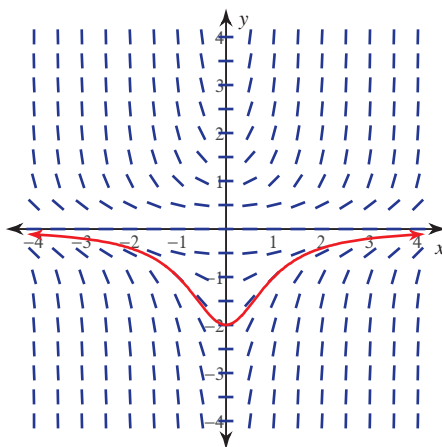
7) $\dfrac{dy}{dx} = 2e^{x-y}$, $y(1) = \ln(2e+1)$

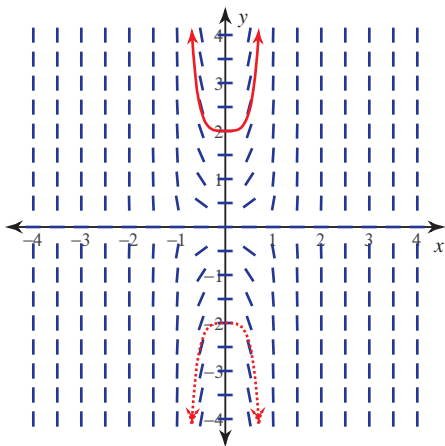$e^y = 2e^x + 1$

$y = \ln\left(2e^x + 1\right)$

8) $\dfrac{dy}{dx} = xy^2$, $y(2) = -\dfrac{2}{5}$

$-\dfrac{1}{y} = \dfrac{x^2}{2} + \dfrac{1}{2}$

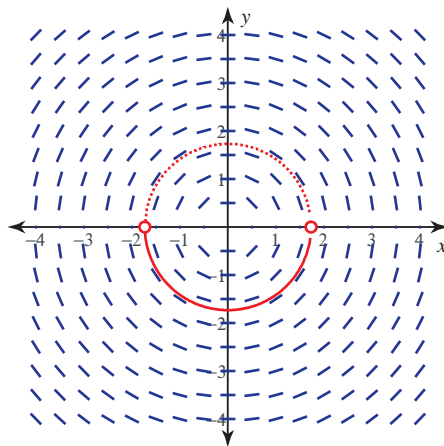$y = -\dfrac{2}{x^2 + 1}$

9) $\dfrac{dy}{dx} = 12x^3 y$, $y(0) = 2$

$\ln\left|y\right| = 3x^4 + \ln 2$

$y = 2e^{3x^4}$

10) $\dfrac{dy}{dx} = -\dfrac{x}{y}$, $y(1) = -\sqrt{2}$

$\dfrac{y^2}{2} = -\dfrac{x^2}{2} + \dfrac{3}{2}$

$y = -\sqrt{-x^2 + 3}$, $-\sqrt{3} < x < \sqrt{3}$